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Blow-up phenomena and global existence for a two-component Camassa–Holm system with an arbitrary smooth function

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ABSTRACT

In the paper, we mainly investigate blow-up phenomena and global existence of strong solutions to a two-component Camassa–Holm systems with an arbitrary smooth function H. For three types of smooth functions H, by using a conservation law and the sign-preserving property of strong solutions, we obtain two new blow-up results and a new global existence result for the two-component system. Our obtained results generalize and cover the recent results in Yan et al. (2015), Zhang and Yin (2015, 2016).

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1. Introduction

Recently, the following multi-component system was introduced in [30]:

$$\begin{cases} m_{j,t} = (m_jH)_x + m_jH + \dfrac{1}{(N+1)^2}\displaystyle\sum_{i=1}^{N}[m_i(u_j - u_{j,x})(v_i + v_{i,x}) + m_j(u_i - u_{i,x})(v_i + v_{i,x})], \\ n_{j,t} = (n_jH)_x - n_jH - \dfrac{1}{(N+1)^2}\displaystyle\sum_{i=1}^{N}[n_i(u_i - u_{i,x})(v_j + v_{j,x}) + n_j(u_i - u_{i,x})(v_i + v_{i,x})], \\ m_j = u_j - u_{j,xx}, \qquad n_j = v_j - v_{j,xx}, \quad 1 \le j \le N, \end{cases} \tag{1.1}$$

where H is an arbitrary smooth function of u_j, v_j, $1 \le j \le N$ and their derivatives.

The system (1.1) was shown to admit Lax pair and infinitely many conservation laws. The bi-Hamiltonian structures and peakon solutions were obtained for some special choices of H. See [30,31] for more details. If H is a polynomial of u_j, v_j, $1 \le j \le N$ and their derivatives, the local well-posedness for (1.1) in Besov spaces, blow-up criteria and Gevrey regularity were studied in [35,24].

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Since the system (1.1) contains an arbitrary function H, it contains actually a large number of multi-component equations. For example, as $N = 1$, $v_1 = 2$, $H = -u_1$, this system is reduced to the well-known Camassa–Holm (CH) equation:

$$m_t + 2mu_x + m_x u = 0, \quad m = u - u_{xx}. \tag{1.2}$$

The CH equation can be regarded as a shallow water wave equation [4,14]. It is completely integrable [3,6,13,15]. It has a bi-Hamiltonian structure [20], and has peakon solutions of the form $ce^{-|x-ct|}$ with $c > 0$, which are orbitally stable [17]. It is worth mentioning that the peakons are suggested by the form of the Stokes water wave of greatest height, see discussions in [7,8,11,12,26,29]. The local well-posedness for the Cauchy problem of the CH equation was studied in [9,18]. Blow-up phenomena and global existence of strong solutions were presented in [5,9,10]. The global weak solutions [16,32], conservative and dissipative solutions were discussed in [1,2] respectively.

As $N = 1$, $v_1 = 2u_1$, $H = -(u_1^2 - u_{1,x}^2)$, the system (1.1) becomes the cubic Camassa–Holm equation:

$$m_t + [m(u^2 - u_x^2)]_x = 0, \quad m = u - u_{xx}, \tag{1.3}$$

which was presented independently by Fokas [19], Fuchssteiner [22], Olver and Rosenau [27], and Qiao [28] as an integrable peakon equation with cubic nonlinearity. The Lax pair, peakon solutions, local well-posedness and blow-up phenomena of (1.3) have been studied in [28,21,23,25].

In this paper, our aim is to investigate blow-up phenomena and global existence of strong solutions to the system (1.1) with $N = 1$. More precisely, that is the following two-component Camassa–Holm equation with an arbitrary smooth function H:

$$\begin{cases} m_t = (mH)_x + mH + \frac{1}{2}m(u - u_x)(v + v_x), \\ n_t = (nH)_x - nH - \frac{1}{2}n(u - u_x)(v + v_x), \\ m = u - u_{xx}, \qquad n = v - v_{xx}, \end{cases} \tag{1.4}$$

where H is an arbitrary smooth function of u, v and their derivatives.

For the cases

$$H = 0, \qquad H = -\frac{1}{2}(u - u_x)(v + v_x), \qquad H = -\frac{1}{2}(uv - u_x v_x), \qquad H = -\frac{1}{2}(uv_x - u_x v), \tag{1.5}$$

blow-up phenomena or global existence of strong solutions to (1.4) were studied in [35,34,33].

Except the above cases (1.5), it is not clear whether or not strong solutions to the system (1.4) with more general smooth functions H may blow up in finite time or exist globally in time. In the paper, we will investigate blow-up phenomena and global existence of strong solutions to the system (1.4) with three types of smooth functions H. As we know, the key step is to estimate the L^∞-norm of u, v, u_x, v_x. We observe that the system (1.4) has a conservation law:

$$\int_{\mathbb{R}} m(v + v_x)dx = \int_{\mathbb{R}} n(u - u_x)dx, \tag{1.6}$$

which holds true for any arbitrary smooth functions H. Then by taking advantage of this conservation law, we observe that for three types of smooth functions H, we can get upper bounds for the L^∞-norm of u, v, u_x, v_x in finite time, under some suitable sign condition assumption on the initial data. Thus it is successful for us to acquire two new blow-up results and a new global existence result as in [35,34,33]. Our obtained results generalize and cover the recent results in [35,34,33].

The rest of this paper is organized as follows. In Section 2, we mainly prove the conservation law (1.6) and the sign-preserving property of strong solutions. In Section 3, we prove some needed estimates and then provide sufficient conditions for strong solutions to blow up in finite time or exist globally in time.

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