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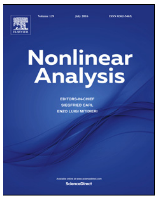

Weak and variational entropy solutions to the system describing steady flow of a compressible reactive mixture

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ABSTRACT

We consider a system of partial differential equations which describes steady flow of a compressible heat conducting chemically reacting gaseous mixture. We extend the result from Giovangigli et al. (2015) in the sense that we introduce the variational entropy solution for this model and prove existence of a weak solution for $\gamma > \frac{4}{3}$ and existence of a variational entropy solution for any $\gamma > 1$. The proof is based on improved density estimates.

1. Introduction

Chemically reacting mixtures appear in many real-life situations, especially in chemical engineering [18], combustion [20], description of some atmospheric phenomena [19] and many others. There are many models of mixtures which can be derived from different general physical models depending on the phenomena which we want to study. We may start from molecular theories like the kinetic theory, statistical mechanics and thermodynamics or from the macroscopic theories like continuum physics and continuum thermodynamics.

Here, we rely on the latter. We continue the program started in [12] which was applied to a special situation for the steady problem in [6].

More precisely, we investigate a system of partial differential equations describing steady flow of chemically reactive, heat conducting, gaseous mixture. The system, which composes of the steady compressible

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Navier–Stokes–Fourier system coupled with the balance of mass fractions, reads

$$\begin{gathered}\operatorname{div}(\varrho\mathbf{u})=0,\\ \operatorname{div}(\varrho\mathbf{u}\otimes\mathbf{u})-\operatorname{div}\mathbb{S}+\nabla\pi=\varrho\mathbf{f},\\ \operatorname{div}(\varrho E\mathbf{u})+\operatorname{div}(\pi\mathbf{u})+\operatorname{div}\mathbf{Q}-\operatorname{div}(\mathbb{S}\mathbf{u})=\varrho\mathbf{f}\cdot\mathbf{u},\\ \operatorname{div}(\varrho Y_k\mathbf{u})+\operatorname{div}\mathbf{F}_k=m_k\omega_k,\quad k\in\{1,\dots,n\}.\end{gathered}\tag{1}$$

In the above equations $\mathbb{S}$ denotes the viscous part of the stress tensor, π the internal pressure of the fluid, $\mathbf{f}$ the external force, E the specific total energy, $\mathbf{Q}$ the heat flux, ω_k the molar production rate of the kth species, $\mathbf{F}_k$ the diffusion flux of the kth species and m_k the molar mass of the kth species which we assume to be equal, hence, without loss of generality

$$m_1=\cdots=m_n=1.\tag{2}$$

System (1) is supplemented by the no-slip boundary conditions for the velocity

$$\mathbf{u}|_{\partial\Omega}=\mathbf{0},\tag{3}$$

together with

$$\mathbf{F}_k\cdot\mathbf{n}|_{\partial\Omega}=0,\tag{4}$$

and the Robin boundary condition for the heat flux

$$-\mathbf{Q}\cdot\mathbf{n}+L(\vartheta-\vartheta_0)=0\tag{5}$$

which means that the heat flux through the boundary is proportional to the difference of the temperature inside Ω and the known external temperature ϑ_0. The coefficient L describes thermal insulation of the boundary and for simplicity we assume it to be constant. We further prescribe the total mass of the mixture

$$\int_\Omega\varrho\,\mathrm{d}x=M>0.\tag{6}$$

The mass fractions $Y_k, k\in\{1,\dots,n\}$, are defined by $Y_k=\frac{\varrho_k}{\varrho}$. Thus, by definition, they satisfy

$$\sum_{k=1}^n Y_k=1.\tag{7}$$

Concerning the chemical production rates, we assume them to be sufficiently regular, bounded functions of ϱ,ϑ and Y_k such that

$$\omega_k\geq 0\quad\text{for } Y_k=0.$$

We also assume

$$\omega_k\geq -CY_k^r\quad\text{for some } C,r>0,\tag{8}$$

which means that a species cannot decrease faster than proportionally to some positive power of its fraction (a possible natural choice is $r=1$). The stress tensor $\mathbb{S}$ is given by the Newton rheological law as

$$\mathbb{S}=\mathbb{S}(\vartheta,\nabla\mathbf{u})=\mu\Big[\nabla\mathbf{u}+(\nabla\mathbf{u})^t-\frac{2}{3}\operatorname{div}\mathbf{u}\mathbb{I}\Big]+\nu(\operatorname{div}\mathbf{u})\mathbb{I},\tag{9}$$

where $\mu=\mu(\vartheta)>0,\ \nu=\nu(\vartheta)\geq 0$, Lipschitz continuous functions in $\mathbb{R}^+$, are the shear and bulk viscosity coefficients, respectively, on which we assume

$$\underline{\mu}(1+\vartheta)\leq\mu(\vartheta)\leq\overline{\mu}(1+\vartheta),\quad 0\leq\nu(\vartheta)\leq\overline{\nu}(1+\vartheta)\tag{10}$$

for some positive constants $\underline{\mu},\overline{\mu},\overline{\nu}$, and $\mathbb{I}$ is the identity matrix.

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