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Interaction of human migration and wealth distribution

V. Volpert[a,b,*], S. Petrovskii[c], A. Zincenko[c]

[a] *Institut Camille Jordan, UMR 5208 CNRS, University Lyon 1, 69622 Villeurbanne, France*
[b] *RUDN University, ul. Miklukho-Maklaya 6, Moscow, 117198, Russia*
[c] *Department of Mathematics, University of Leicester, Leicester, LE1 7RH, UK*

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ABSTRACT

Dynamics of human populations depends on various economical and social factors. Their migration is partially determined by the economical conditions and it can also influence these conditions. This work is devoted to the analysis of the interaction of human migration and wealth distribution. The model consists of a system of equations for the population density and for the wealth distribution with conventional diffusion terms and with cross diffusion terms describing human migration determined by the wealth gradient and wealth flux determined by human migration. Wealth production and consumption depend on the population density while the natality and mortality rates depend on the level of wealth. In the absence of cross diffusion terms, dynamics of solutions is described by travelling wave solutions of the corresponding reaction–diffusion systems of equations. We show persistence of such solutions for sufficiently small cross diffusion coefficients. This result is based on the perturbation methods and on the spectral properties of the linearized operators.

1. Introduction

Dynamics of human population has long been a focus of research and controversy. Early insights into its uncontrolled growth resulted in a rather negative outlook and even disastrous conclusions because of the finiteness of available resources [14] (see also Section 1.1 in [18]). Although nowadays this gloomy forecast is not widely shared, good understanding of the factors controlling the growth and proliferation of human population is required in order to avoid crises, poverty and social tensions.

Due to the effect of cultural, religious, political and other phenomena specific for human societies, e.g. see [23], the population dynamics of humans is apparently much more complicated than the dynamics of animal species. Mathematical models of human dynamics allowing for the specifics and complexity of social

* Corresponding author at: Institut Camille Jordan, UMR 5208 CNRS, University Lyon 1, 69622 Villeurbanne, France. Fax: +33 472431687.
E-mail address: volpert@math.univ-lyon1.fr (V. Volpert).

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and cultural interactions are at their infancy. There are, however, factors and processes that are important for all living beings, and one of them is movement. As well as animal species, the dynamics of human population has a distinct spatial aspect. In particular, human migration is a ubiquitous phenomenon that has been shaping countries and societies over centuries of human history [24,5] and that takes place at various scales ranging from individuals to nations [9,20]. The direction of migration (or, more generally, population flow) is often determined by the availability of resources and/or by the quality of living.[1] That can be thought about as the effect of the resource gradient. But the effect of resource gradient is known to be a factor that affects invasion (or, more generally, dispersal) of animal and plant species too, e.g. see [12,11]. Therefore, mathematical approaches that were developed to describe spatiotemporal dynamics of ecological populations [18,13] can be expected to be a reasonable starting point to describe the human dynamic too.

Proliferation of biological populations in space occurs due to an interplay between their reproduction and the movement of individuals [22,7,21]. The density $p(x,t)$ of the population which depends on the space x and on time t can be calculated from a mathematical model. Nowadays, a broad variety of models can be used for this purpose [10] depending on the properties of the environment, e.g. uniform or fragmented [16], and on the type of individual movement, e.g. ordinary diffusion (Brownian motion) or anomalous diffusion [15]. One commonly used type of model is given by the reaction–diffusion equations [27]. In the case where only a single species is considered explicitly, the model consists of a single partial differential equation:

$$\frac{\partial p}{\partial t} = D\frac{\partial^2 p}{\partial x^2} + f(p), \tag{1.1}$$

where the diffusion term corresponds to random movement of the individuals (ordinary diffusion), and the reaction term represents their natality and mortality. This equation with the function $f(p) = p(1-p)$ was introduced in [4] in order to describe propagation of the dominant gene. In a more general ecological context, $f(p) = \alpha p(K-p)$ where α is the per capita population growth rate and K is the carrying capacity of the environment [18].

A remarkable property of Eq. (1.1) is that, if considered in an unbounded uniform space, for a broad class of initial conditions its large-time asymptotical solution is given by a travelling wave, that is by the solution $p(x,t) = P(x-ct)$, where $c = (2D\alpha)^{1/2}$ is the speed of propagation [8]. Existence, stability and the speed of propagation of travelling waves are studied in detail for the logistic function $f(p)$ and for more general nonlinearities (see [27,29] and the references therein).

Considering Eq. (1.1) in the context of population dynamics, it therefore predicts that the propagation of the population is described by a travelling wave with the constant speed c determined by the coefficients of the equation. The properties of the propagation becomes more complicated if other species (e.g. a predator or pathogen or competitor) or components are included (e.g. resource); the model is then given by a systems of PDEs. Well known examples are given by a system of competing species and by the prey–predator system where a simple travelling population front may turn into a sequence of fronts [19] or even a more complicated pattern of spread [27,17].

In the next section we will introduce the model of economical migration studied in this work. It consists of a system of two equations with cross diffusion terms. In the case of zero cross diffusion coefficients, it reduces to a conventional reaction–diffusion system of equations. We will study the case of small cross diffusion coefficients and will show that migration of the population can be described by travelling wave solutions.

[1] Considering, for instance, the recent surge in the migration to Europe, whilst some of the migrants were apparently fleeing war or genocide, many migrants openly stated that they were in search of economic benefits and more comfortable life.

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