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Note on a two-species competition-diffusion model with two free boundaries☆

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ABSTRACT

In Guo and Wu (2015) and Wu (2015), the authors studied a two-species competition-diffusion model with two free boundaries. These two free boundaries describing the spreading fronts of two competing species, respectively, may intersect each other as time evolves. The existence, uniqueness and long time behavior of global solution have been established. In this note we discuss the conditions for spreading and vanishing, and more accurate limits of (u, v) as $t \to \infty$ when spreading occurs. Some new results and simpler proofs will be provided.

1. Introduction

Recently, Guo & Wu [3] studied the existence and uniqueness of global solution (u, v, s_1, s_2) to the following free boundary problem

$$\begin{cases} u_t - d_1 u_{xx} = r_1 u(1 - u - kv), & t > 0,\ 0 < x < s_1(t), \\ v_t - d_2 v_{xx} = r_2 v(1 - v - hu), & t > 0,\ 0 < x < s_2(t), \\ u_x(t, 0) = v_x(t, 0) = 0, & t \geq 0, \\ s_1'(t) = -\mu_1 u_x(t, s_1(t)), \quad s_2'(t) = -\mu_2 v_x(t, s_2(t)), & t \geq 0, \\ u = 0 \quad \text{for } x \geq s_1(t), \quad v = 0 \quad \text{for } x \geq s_2(t), & t \geq 0, \\ u(0, x) = u_0(x), \quad v(0, x) = v_0(x), & x \in [0, \infty), \\ s_1(0) = s_1^0 > 0, \quad s_2(0) = s_2^0 > 0, \end{cases}$$

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where parameters are all positive constants, and $u_0(x), v_0(x)$ satisfy

$$\begin{array}{llll} u_0 \in C^2([0, s_1^0]), & u_0'(0) = 0, & u_0(x) > 0 \quad \text{in } [0, s_1^0), & u_0(x) = 0 \quad \text{in } [s_1^0, \infty), \\ v_0 \in C^2([0, s_2^0]), & v_0'(0) = 0, & v_0(x) > 0 \quad \text{in } [0, s_2^0), & v_0(x) = 0 \quad \text{in } [s_2^0, \infty). \end{array}$$

Furthermore, Guo & Wu [3] and Wu [10] investigated the conditions of spreading and vanishing, and more accurate limits of (u, v) as $t \to \infty$ when spreading occurs for the cases $0 < k < 1 < h$ and $0 < k, h < 1$, respectively. In this model, $x = s_1(t)$ and $x = s_2(t)$ describe the spreading fronts of two competing species, respectively, and may intersect each other as time evolves.

By use of the arguments of [7, Theorem 2.1] or [12, Lemma 3.1] we can prove that $s_1'(t), s_2'(t) > 0$, and

$$(u, v, s_1, s_2) \in C^{1+\frac{\alpha}{2}, 2+\alpha}(\mathcal{D}_\infty^{s_1}) \times C^{1+\frac{\alpha}{2}, 2+\alpha}(\mathcal{D}_\infty^{s_2}) \times \left[C^{1+\frac{1+\alpha}{2}}([0, \infty))\right]^2,$$

where $\mathcal{D}_\infty^{s_i} = \{t > 0,\ 0 \le x \le s_i(t)\}$. Moreover, if $s_1^\infty = \lim_{t\to\infty} s_1(t) < \infty$, then there exists a positive constant C such that

$$\|u(t, \cdot)\|_{C^1([0,\, s_1(t)])} \le C, \quad \forall\, t \ge 1; \qquad \|s_1'\|_{C^{\alpha/2}([1,\infty))} \le C. \tag{1}$$

Similarly, if $s_2^\infty = \lim_{t\to\infty} s_2(t) < \infty$, then there exists a positive constant C' such that

$$\|v(t, \cdot)\|_{C^1([0, s_2(t)])} \le C', \quad \forall\, t \ge 1; \qquad \|s_2'\|_{C^{\alpha/2}([1,\infty))} \le C'. \tag{2}$$

When $s_1^\infty = \infty$ (resp., $s_2^\infty = \infty$), we say that the species u (resp., v) spreads successfully. When $s_1^\infty < \infty$ and $\lim_{t\to\infty} \max_{[0,s_1(t)]} u(t, \cdot) = 0$ (resp., $s_2^\infty < \infty$ and $\lim_{t\to\infty} \max_{[0,s_2(t)]} v(t, \cdot) = 0$), we say that the species u (resp., v) vanishes.

We still study the conditions for spreading and vanishing, and more accurate limits of (u, v) as $t \to \infty$ when spreading occurs. Some new results and simpler proofs are provided. This short paper can be considered as the supplements of papers [3,10].

2. Preliminaries

Proposition 1 (*[8, Proposition 2.1]*)**.** *Let d, r, a be fixed positive constants. For any given $\varepsilon, L > 0$, there exists $l_\varepsilon > \max\{L, \frac{\pi}{2}\sqrt{d/(ra)}\}$ such that, when a non-negative $C^{1,2}$ function z satisfies*

$$\begin{cases} z_t - dz_{xx} \ge rz(a - z), & t > 0,\ 0 < x < l_\varepsilon, \\ z_x(t, 0) = 0, \quad z(t, l_\varepsilon) \ge 0, & t \ge 0 \end{cases}$$

and $z(0, x) > 0$ in $(0, l_\varepsilon)$, then $\liminf_{t\to\infty} z(t, x) \ge a - \varepsilon$ uniformly on $[0, L]$.

Proposition 2 (*[1,2]*)**.** *For any given $d, a, b, \mu > 0$, the problem*

$$\begin{cases} dq'' - cq' + q(a - bq) = 0, \quad 0 < y < \infty, \\ q(0) = 0, \quad q'(0) = c/\mu, \quad q(\infty) = a/b, \\ c \in (0, 2\sqrt{ad}); \quad q'(y) > 0, \quad 0 < y < \infty \end{cases} \tag{3}$$

has a unique solution (q, c). Denote $\gamma = (\mu, a, b, d)$ and $c = c(\gamma)$. Then $c(\gamma)$ is strictly increasing in μ and a, respectively, and is strictly decreasing in b. Moreover,

$$\lim_{\frac{a\mu}{bd}\to\infty} \frac{c(\gamma)}{\sqrt{ad}} = 2, \qquad \lim_{\frac{a\mu}{bd}\to 0} \frac{c(\gamma)}{\sqrt{ad}} \frac{bd}{a\mu} = \frac{1}{\sqrt{3}}. \tag{4}$$

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