ARTICLE IN PRESS

Nonlinear Analysis ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Nonlinear Analysis

www.elsevier.com/locate/na

Discontinuous traveling wave entropy solutions for a sedimentation–consolidation model

Tianyuan Xu [a], Jingxue Yin [a], Shanming Ji [b,*]

[a] School of Mathematical Sciences, South China Normal University, Guangzhou 510631, PR China
[b] School of Mathematics, South China University of Technology, Guangzhou 510640, PR China

ARTICLE INFO

Article history:
Received 8 June 2016
Accepted 29 November 2016
Communicated by S. Carl

Keywords:
Sedimentation–consolidation model
Discontinuous traveling wave
Entropy solution

ABSTRACT

This paper deals with a mathematical model of gravitational sedimentation–consolidation processes, for which the existence of discontinuous traveling wave entropy solutions is established by using the phase plane analysis method.

1. Introduction

In this paper, we consider discontinuous traveling wave entropy solutions for a mathematical model of gravitational sedimentation–consolidation processes governed by the following degenerate quasilinear parabolic equation

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} f(u) = \frac{\partial}{\partial x}\left(a(u)\frac{\partial u}{\partial x}\right) + h(u), \quad x \in \mathbb{R},\ t > 0, \tag{1}$$

where $u = u(x,t) \in (0,1)$ is the volumetric solids concentration, $f(s), h(s)$ are appropriately smooth functions and $a(s) \geq 0$ is discontinuous.

The phenomenological theory of sedimentation–consolidation processes is widely used in chemical engineering, mineral processing and wastewater treatment for the solid–liquid separation of suspensions. It was Kynch [15] who first proposed the kinematical sedimentation theory. Studies [17,8,5,11] of the sedimentation–consolidation process are mostly based on Kynch's theory, which describes the batch settling

* Corresponding author.
E-mail address: jishanming7@126.com (S. Ji).

http://dx.doi.org/10.1016/j.na.2016.11.021
0362-546X/

Please cite this article in press as: T. Xu, et al., Discontinuous traveling wave entropy solutions for a sedimentation–consolidation model, Nonlinear Analysis (2016), http://dx.doi.org/10.1016/j.na.2016.11.021

by the conservation law

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0.$$

Only a few works [2,6,7] are related to the model with a strongly degenerate diffusion so far.

In a recent series of papers, the traveling waves of balance law were paid more attention. For example, if $a(u) \equiv 0$, (1) reduces to the scalar hyperbolic balance law

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} f(u) = h(u), \quad x \in \mathbb{R},\ t > 0, \tag{2}$$

which is extensively studied, for example, in [12,18,20]. It was Fan and Hale [12] who first studied the discontinuous traveling waves for (2). Sinestrari [20] studied related properties of such traveling waves. It is well known that the effects of viscosity on the balance law should be included in many practical problems such as wastewater treatment [6], the model of car traffic flow on a highway [23], etc. In the case of $a(u) \not\equiv 0$, (1) becomes the scalar parabolic balance law, to which a few important research works [10,9] have been devoted in the past several decades. When $a(u) \equiv \varepsilon > 0$, Y. Wu and X. Xing [21] considered the traveling waves of the following scalar parabolic balance law

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} f(u) = \varepsilon \frac{\partial^2 u}{\partial x^2} + h(u), \quad x \in \mathbb{R},\ t > 0. \tag{3}$$

Owing to the linear parabolicity, the previous works mentioned above admit only smooth traveling waves. However, discontinuous surfaces have been observed in some viscous balance phenomena [7].

As far as we know, it is worthy of noticing that the existing results of discontinuous traveling waves are mainly limited to the hyperbolic equations. Jordan [13] modified the usual Ficken-based constitutive relation for traffic flux and found that shock waves are possible only if the diffusivity is non-vanishing. Using phase plane analysis, Landman et al. [16] investigated the smooth and discontinuous traveling waves of a model of chemotactic cell migration. Recently, a class of hyperbolic reaction–diffusion models were proposed by Barbera et al. [3] and the existence of discontinuous traveling waves were proved theoretically and numerically.

In the present paper, we pay our attention to studying the traveling wave entropy solutions for the modified sedimentation–consolidation model proposed in [7]. Completely different from the hyperbolic balance law [20] ($a(s) \equiv 0$) and the strongly degenerate parabolic balance law [24] ($a(s) \in C^1$), we prove the nonexistence of continuous traveling wave entropy solution for (1) with discontinuous $a(s)$. Therefore, the previous method for constructing discontinuous traveling waves by continuous traveling wave entropy solution is inapplicable. It is worth mentioning that the entropy condition involving diffusion differs from that in fully degenerate case [20,24], since the diffusion related entropy condition provides the local monotonicity.

This paper is organized as follows. In Section 2, the mathematical model is derived. In Section 3, after introducing some notations and the definition of entropy solutions derived from the vanishing viscosity method, we present some auxiliary lemmas and state the main results. Subsequently, in Section 4, the nonexistence of continuous traveling wave entropy solution, the existence and uniqueness of discontinuous traveling wave entropy solution, are proved.

2. Mathematical model

In the present model, the corresponding governing equations can be derived following the classical theory of hydrodynamics. In terms of local mean variables, the continuity equations [1] for the solids and the fluid