ARTICLE IN PRESS

Nonlinear Analysis ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Nonlinear Analysis

www.elsevier.com/locate/na

Editorial

The various stages of Nicola Fusco

Nicola Fusco is an outstanding analyst and a leading expert in the Calculus of Variations. This special issue of Nonlinear Analysis collects a series of papers on topics that are close to his research interests and it is dedicated to him on the occasion of his 60th birthday.

Nicola was born in Naples on August 14, 1956, and studied at the University of Naples "Federico II" under the guidance of Carlo Sbordone. He continued as a research fellow at CNR for three years, then becoming an assistant and visiting professor at the Universities of Naples and at the Australian National University at Canberra. He was promoted to full professor of Mathematics in 1987. Since then Nicola has taught at the universities of Salerno, Naples, Florence.

For his remarkable research achievements, Nicola has gained several recognitions. Amongst them we highlight the prestigious Caccioppoli Prize in 1994, the Tartufari Prize in 2010, and the election to the Accademia dei Lincei in the same year. He has been an invited speaker both at the ICM (2010) and at the ECM (2008). He has been appointed Finnish Distinguished Professor in 2010. Nicola has been recently elected member of the Executive Committee of the European Mathematical Society.

Nicola's scientific production is multifaceted, and in almost forty years of activity has spanned several fundamental aspects of the Calculus of Variations and related fields. Some of his results are by now classical, they had an enormous impact, in turn generating a large literature devoted to their analysis and development. It is therefore not easy to give a full account of Nicola's remarkable accomplishments, but we will attempt an overview.

1. The early Nicola

Nicola's early interests were devoted to the theory of Gamma-convergence, homogenization and, mostly, to lower semicontinuity problems in the Calculus of Variations. One of his early works [24] deals exactly with the lower semicontinuity of so-called quasiconvex integral functionals. Quasiconvexity has been introduced by Morrey [35], and later widely developed by Ball [8] in the context of Nonlinear Elasticity. This concept, of fundamental nature, is aimed at providing a necessary and sufficient condition for the lower semicontinuity of integral functionals of the Calculus of Variations in the vectorial case, thereby addressing the fundamental problem of existence via the so-called Direct Methods. What Nicola proved in [24] is the lower semicontinuity of such integrals in a topology which is still slightly stronger than the natural one (namely, $W^{1,p+\delta}$ versus the natural topology, that is $W^{1,p}$, for $p > 1$, with p being the order of the polynomial bounding the integrand from above). Although this result is still sufficient to get existence of minimizers when applied to minimizing sequences (via Ekeland lemma, as shown in [34]), the natural open problem of proving lower semicontinuity in the natural topology remained as one of the most outstanding in the Calculus of Variations of that time. This issue was finally settled in the groundbreaking paper [1], written with his friend and collaborator

http://dx.doi.org/10.1016/j.na.2016.08.025

Please cite this article in press as: B. Dacorogna, et al., The various stages of Nicola Fusco, Nonlinear Analysis (2016), http://dx.doi.org/10.1016/j.na.2016.08.025

Emilio Acerbi, then at Scuola Normale at Pisa and nowadays at Parma. It is not easy to describe the incredible impact that this paper has had on modern Nonlinear Analysis, not only for the result itself but also, and mostly, for the innovative techniques it featured. Indeed, in order to overcome the lack of uniform higher integrability assumed in [24], a very intricate combination of new tools such as the Biting Lemma and Lipschitz extensions was developed and applied in this context, finally allowing for a detailed study of concentration effects in minimizing sequences. The tools and the viewpoints developed in [1] have since then become classical, influencing entire generations of mathematicians working in the field of the Calculus of Variations.

With the impact of [1] gaining both Emilio and Nicola an instant international reputation, Nicola decided to switch to a different topic, namely regularity theory. A brave choice, as this is usually considered a highly technical and difficult subject. Nicola has since changed his field of investigation often, and this was just the first time this happened, as we will see later. The occasion came with the invitation of Neil Trudinger to the Australian National University as visiting professor. Before moving to Canberra, Nicola married his beloved wife Tonia Tschantret, an exceptional and beautiful woman who has been since then an invaluable and loving source of support for Nicola. They have three children, Nicoletta, Andrea and Marco.

In Canberra, Nicola started learning and working on regularity theory, thereby initiating a fruitful collaboration with John E. Hutchinson, the widely known author of one of the most influential papers on the theory of fractal sets [33]. They were both newcomers to regularity theory. This joint work led to a series of outstanding regularity papers. In particular, we mention the first one [25], where a more general, non-autonomous version of a preceding partial regularity theorem for minimizers of quasiconvex functionals of Evans [20] is extended. We also recall [26]. This remarkable and pioneering paper is the first one where partial regularity theorems for minima of so-called polyconvex functionals are given; completely new methods for handling the special and apparently untreatable difficulties inherent to polyconvexity are introduced there. In time, Nicola completed the partial regularity theory of quasiconvex functionals treating the problem under very general assumptions [2,11].

Regularity theory has since been one of the main fields of investigation of Nicola. In this respect, he has written a series of papers on minimizers of functionals with so-called non-standard growth conditions. These are functionals whose integrands are characterized by the fact of having growth conditions exhibiting different rates from above and below. As such they are therefore automatically linked to non-uniformly elliptic operators when considering their Euler–Lagrange equations and can even be of non-polynomial type. In particular, together with Carlo Sbordone, Nicola has written a very influential paper [31] on the validity of the so-called Gehring lemma and higher integrability of minima for such functionals. Related to higher integrability, Carlo and Nicola have also obtained a few remarkable results on limiting integrability problems related to non-negative Jacobians [10] with Haïm Brezis, and Sobolev embedding theorem [27], with Pierre Louis Lions. Moreover, again with Acerbi, Nicola has published a pioneering paper on functionals with variable growth exponent [3]. This is a special subclass of functionals with nonstandard growth that has first been considered by Zhikov [37] in the context of homogenization. A few years later such types of functionals became extremely popular, attracting the interest of very many researchers [18]. The paper [3] has been one of the very first in this direction.

2. The intermediate Nicola

At the beginning of the 90s Nicola got interested in the so-called free discontinuity problems. These problems involve functionals whose energy is a combination of both volume and perimeter terms, i.e., terms with different dimensions and scaling properties both concur in the penalization process. In the original formulation of the problem, the competitors are not functions, but rather couples of piecewise C^1-functions with a related $n-1$-dimensional set of discontinuities. These functionals naturally arise in the study of image

Please cite this article in press as: B. Dacorogna, et al., The various stages of Nicola Fusco, Nonlinear Analysis (2016), http://dx.doi.org/10.1016/j.na.2016.08.025