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Estimation of wave heights from pressure data at the bed in the presence of uniform underlying currents

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ABSTRACT

We provide some bounds on the estimated wave heights (valid for all ranges of amplitudes) from pressure measurements at the flat bed for steady water waves flowing over underlying uniform currents. The derived upper bound for the wave height for the case when the speed of the underlying current is greater than the wave speed is different from the one when there is no current. The results in this paper also confirm that waves of nontrivial amplitudes cannot exist where the speed of the underlying current is same as the wave speed, a result that was previously proved by Basu (2016) in a different context following another approach.

1. Introduction

Wave heights play an important role in estimating forces on maritime and offshore structures, estimating power generated by wave energy devices and designing possible means of flood control at coastal regions. Observations of sea waves can be either done remotely by satellite measurements or by in situ measurements. Remote-sensing techniques being an indirect way of measurement require ground-truth verification. Recovery of wave heights from pressure data at the bed is a practical means to estimate wave heights from in-situ measurements which can be also used for such verifications. Use of pressure sensors among other types of sensors is widespread in practice due to their low costs, ease of installation and relatively less liability in the event of damage [18]. Pressure sensors are usually deployed underwater at fixed locations on the rigid bed. These sensors provide accurate measurement of the pressure at the bed, but recovery of wave heights from these pressure measurements requires some additional considerations to unfold the pressure–wave height relationship.

Researchers in the past have developed expressions for recovering the wave height from the pressure at the bed, e.g. [20] used the hydrostatic formula for tsunami detection and [16] developed explicit expressions for pressure and surface waves in the setting of linear wave theory. However, these studies either did not

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consider the wave effects or are valid for waves of small amplitude. In fact, linear theory may overestimate wave heights by over 10% even for waves of moderate amplitude when compared with observed data [22]. Since most waves relevant in ocean engineering do not fall into the category of small amplitude theory, it is necessary to consider the governing equations without approximations. Researchers such as [20,15,7,4,3,11] and [12] have pursued further investigations in this direction. Considering the presence of vorticity, Henry [17] showed that pressure function on the flat bed prescribes a unique surface profile for the resulting solitary waves. The conclusion derived from these investigations was that an exact recovery of the wave profile from the pressure at the flat bed is indeed possible. However, in this context the only explicit results for the recovery of surface profile is due to [7] for solitary waves while others such as [4] and [3] require either the solution of either a non-linear differential equation or an implicit functional equation for periodic travelling waves. Constantin [10] recently developed another approach to provide simple, explicit but accurate estimates of wave heights for the fully nonlinear large amplitude 2D travelling gravity waves which differed from some of the previously proposed approaches and results.

Inspite of several results available on the wave height recovery problem, simple and accurate expressions for the recovery of wave heights from the pressure data at the bed for the case of large amplitude surface waves with underlying currents, are not available in the literature currently, which is the contribution of this paper. Recently, Basu [1] reported some new results on flow fields and pressure distribution for the case of 2D wave-uniform current interactions using maximum principles for elliptic partial differential equations. This paper further builds upon the work by Basu [1] in exploiting some of those results and extends the work by Constantin [10], by developing bounds on wave heights for 2D periodic travelling waves from pressure data at the bed with wave-uniform current interaction, which are equally valid for all ranges of amplitude (small and large).

2. Basic equations

We consider a 2D flow represented in the Cartesian co-ordinate (X,Y) with the direction of wave propagation along the X-axis and the Y-axis pointing vertically upwards. The depth of water at rest is denoted by $d > 0$ which leads to the equation of the flat bed as $Y = -d$. The free surface is given by $Y = \eta(X - ct)$, where $c > 0$ is the speed of propagation of the periodic travelling waves. The surface waves are assumed to propagate over a uniform underlying current of speed k. The flow is irrotational and it may be noted that the uniform underlying current does not introduce any vorticity to the flow.

We investigate periodic travelling waves of wave length L, with profile η moving at constant speed c. The wave crest is assumed to be located at $X = 0$. Also, let the flow field (u,v) have a space–time dependence of the form $(X - ct)$, i.e. $u := u(X - ct, Y)$ $v := v(X - ct, Y)$. To analyse this problem let us introduce a co-ordinate transformation to a moving frame $x = X - ct, y = Y$. Flows without stagnation points are considered, so that, by regularity results discussed in [13], all functions are real analytic. The only waves that are thus excluded are the so-called Stokes waves of greatest height (see the discussions in [21,8,19]).

The governing equations for the water waves under the inviscid, incompressible conditions are given by the Euler's equation

$$\begin{aligned}(u-c)u_x + vu_y &= -\frac{1}{\rho}P_x \quad \text{for} \quad -d \le y \le \eta(x)\\ (u-c)v_x + vv_y &= -\frac{1}{\rho}P_y - g \quad \text{for} \quad -d \le y \le \eta(x)\end{aligned} \tag{1}$$

where $P(x,y)$ is the hydrodynamic pressure, ρ is the density and g is the acceleration due to gravity.

The mass conservation equation for a homogeneous, incompressible fluid is

$$u_x + v_y = 0 \quad \text{for} \quad -d \le y \le \eta(x) \tag{2}$$