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Removable singularities for degenerate elliptic Pucci operator on the Heisenberg group

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In this paper, we study viscosity solutions to a class of degenerate elliptic Pucci operators modelled on the Heisenberg group, where the second order term is obtained by a composition of degenerate elliptic Pucci operator with the degenerate Heisenberg Hessian matrix. We study and answer the question: Which compact sets have the property that each viscosity subsolution outside this set, which is bounded below, can be extended to a viscosity subsolution on this set.

1. Introduction and main result

In this paper, we are interested in the removable singularities for viscosity subsolutions to degenerate elliptic Pucci operators in the setting of the Heisenberg group.

In order to describe our main result, we will first recall some basic facts and properties of Heisenberg group.

1.1. Heisenberg group

For $n \in \mathbb{N}^+$, let $\mathbb{H}^n$ be the Heisenberg group $(\mathbb{R}^{2n+1}, \circ)$, where $\circ$ is defined as

$$\xi \circ \hat{\xi} := \left(x + \hat{x}, y + \hat{y}, t + \hat{t} + 2\sum_{i=1}^{n} (x_i \hat{y}_i - y_i \hat{x}_i) \right)$$

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for any $\xi = (x, y, t)$, $\hat{\xi} = (\hat{x}, \hat{y}, \hat{t})$ in $\mathbb{H}^n$, with $x = (x_1, \ldots, x_n)$, $\hat{x} = (\hat{x}_1, \ldots, \hat{x}_n)$, $y = (y_1, \ldots, y_n)$ and $\hat{y} = (\hat{y}_1, \ldots, \hat{y}_n)$ denoting elements of $\mathbb{R}^n$. We consider the norm on $\mathbb{H}^n$ defined by

$$\|\xi\|_H := \left[\left(\sum_{i=1}^{n} \left(x_i^2 + y_i^2 \right) \right)^2 + t^2 \right]^{\frac{1}{4}}.$$

The corresponding distance on $\mathbb{H}^n$ is defined accordingly by setting

$$d_H(\xi, \hat{\xi}) := \|\hat{\xi}^{-1} \circ \xi\|_H,$$

where $\hat{\xi}^{-1}$ is the inverse of $\hat{\xi}$ with respect to $\circ$, i.e. $\hat{\xi}^{-1} = -\hat{\xi}$. For every $\xi \in \mathbb{H}^n$ and $R > 0$, we will use the notations

$$D_R(\xi) := \{\eta \in \mathbb{H}^n : d_H(\xi, \eta) < R\}$$

and

$$\bar{D}_R(\xi) := \{\eta \in \mathbb{H}^n : d_H(\xi, \eta) \leq R\}.$$

The vector fields

$$\begin{aligned} X_j &:= \frac{\partial}{\partial x_j} + 2y_j \frac{\partial}{\partial t}, \quad j = 1, \ldots, n, \\ Y_j &:= \frac{\partial}{\partial y_j} - 2x_j \frac{\partial}{\partial t}, \quad j = 1, \ldots, n, \\ T &:= \frac{\partial}{\partial t} \end{aligned}$$

form a base of the Lie algebra of vector fields on the Heisenberg group. The Heisenberg gradient, or horizontal gradient, of a regular function $w : \mathbb{H}^n \to \mathbb{R}^1$ is then defined by

$$\nabla_H w := (X_1 w, \ldots, X_n w, Y_1 w, \ldots, Y_n w)^T,$$

while its Heisenberg Hessian matrix is

$$\nabla_H^2 w := \begin{pmatrix} X_1X_1w & \cdots & X_nX_1w & Y_1X_1w & \cdots & Y_nX_1w \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ X_1X_nw & \cdots & X_nX_nw & Y_1X_nw & \cdots & Y_nX_nw \\ X_1Y_1w & \cdots & X_nY_1w & Y_1Y_1w & \cdots & Y_nY_1w \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ X_1Y_1w & \cdots & X_nY_1w & Y_1Y_1w & \cdots & Y_nY_1w \end{pmatrix}.$$

By direct computation, it is easy to see that

$$\nabla_H^2 w = \frac{1}{2} \left(\nabla_H^2 w + \left(\nabla_H^2 w \right)^T \right) + 2TwJ,$$

where

$$J = \begin{pmatrix} 0_n & I_n \\ -I_n & 0_n \end{pmatrix}.$$

We call $\frac{1}{2}(\nabla_H^2 w + (\nabla_H^2 w)^T)$ the symmetric part of $\nabla_H^2 w$ and denote it by $\nabla_{H,s}^2 w$.