Nonlinear Analysis 162 (2017) 13–21

Contents lists available at ScienceDirect

Nonlinear Analysis

www.elsevier.com/locate/na

Extrema of the dynamic pressure in an irrotational Stokes wave with underlying currents and infinite depth

Lili Fan[a], Hongjun Gao[b,c,*], Lei Mao[b]

[a] *College of Mathematics and Information Science, Henan Normal University, Xinxiang 453007, China*
[b] *School of Mathematical Sciences, Institute of Mathematics, Nanjing Normal University, Nanjing 210023, China*
[c] *Jiangsu Center for Collaborative Innovation in Geographical Information Resource Development and Application, Nanjing Normal University, Nanjing 210023, China*

ARTICLE INFO

Article history:
Received 13 April 2017
Accepted 10 June 2017
Communicated by Enzo Mitidieri

MSC 2010:
35Q35
35J15

Keywords:
Stokes wave
Dynamic pressure
Underlying current
Deep-water

ABSTRACT

In this paper, we investigate the dynamic pressure in an irrotational Stokes wave in flows of finite depth with uniform underlying currents and infinite depth without underlying currents respectively. Using the maximum principles in combination with exploiting some of the physical structures of the problem itself, we prove that, irrespectively of uniform underlying current, infinite depth and wave amplitude, the maximum and minimum of the dynamic pressure are attained at the wave crest and at the wave trough respectively.

1. Introduction

It is known that the linear theory approximation is valid for waves of small amplitude, which however fails for waves of moderate and large amplitude relevant in ocean engineering. This urges researchers to turn their attention to the waves of large amplitude in recent years, among which is the Stokes wave of large amplitude. The rigorous mathematical investigation of the structures and behaviour of a Stokes wave of large amplitude has been carried out by Constantin and the others, including trajectories of particles [7], mean velocities [10], analyticity and symmetry of streamlines [14,16], pressure [18] and so on. Concerning Stokes wave in fluid domain with infinite depth, Henry has investigated the horizontal velocity [22], the trajectories of particles [21] and the pressure [23]. The study of the irrotational Stokes waves with uniform underlying currents is due to Basu [2], where he obtained that when the speed of the current is greater than

* Corresponding author.
E-mail addresses: fanlily89@126.com (L. Fan), gaohj@njnu.edu.cn (H. Gao).

http://dx.doi.org/10.1016/j.na.2017.06.006
0362-546X/

the wave speed, the nature of the flow fields is different from what is expected where no underlying current was considered [7,9,12] and the pressure field is unaffected due to the presence of underlying currents. There are also some other noteworthy work related to this area of nonlinear water waves [25,27,28,31,33].

The dynamic pressure, encoding the fluid motion, is one of the two components of the total pressure beneath a surface water wave and the other is a hydrostatic part that counteracts the force of gravity to make the fluid be in an equilibrium state. The study of the behaviour of the pressure has not only theoretical sense [13,30,32] but also practical applications, such as the estimations of the force acting on maritime structures [4–6,11,19,24,26,29]. Inspired by the recent work of Constantin on the extrema of the dynamic pressure in an irrotational regular wave train [13], we will study the influence of the uniform underlying currents and infinite depth on the extrema of the dynamic pressure in this paper.

In regards to the appearance of uniform underlying currents, three specific situations are considered depending on relative magnitude of the wave speed c and the underlying current strength k. To deal with the case $k > c$, we first eliminate the possibility of the relative mass flux $m \geq 0$ by employing the maximum principle and Hopf's maximum principle, then a reapplication of Hopf's maximum principle on the stream function ψ enables us to get that the horizontal velocity u is greater than the wave speed c, which Basu obtained by analysing the non-dimensional system of the stream function, an equivalent system for the governing equations, in [2]. Next, we take advantage of the maximum principles for elliptic partial differential equations in combination with the techniques used in [13] to obtain the results about the extrema of dynamic pressure. In the case $k = c$, an interesting phenomenon arises as the flow is reduced to a uniform flow with flat surface, eliminating any hydrodynamic structure. Finally, the flow in the case $k < c$ can be treated as a flow with wave speed $\tilde{c} = c - k$ and no current and hence the result of extrema of dynamic pressure can be got in a similar way as in [13]. As an application of the extrema of dynamic pressure, we give an estimation of the lower bound of the maximum elevation, which in accordance with a recent result in [3].

As for the extrema of dynamic pressure in deep water, it is natural to guess that the similar results are valid if we take the limit of the finite depth d in [13] to plus infinity and it is indeed the case. However, the failure of application of the maximum principle in the fluid domain with infinite depth makes the proof not trivial. Based on the results about the total pressure constructed by Henry [23], we eliminate the possibility that the extrema can be obtained in the interior of the domain by assuming the contrary. Then the analysis of the dynamic pressure along the boundary of fluid domain is relatively normal and the results can be followed. The idea presented in [13] has also been extended to deal with the dynamic pressure of a solitary wave recently [20], where the solitary waves can be regarded as the long-wave limit of periodic waves [1], while the flow characteristics beneath the surface, in particular, the particle paths have a quite different behaviour—see the discussion in the papers [7,8,15,17].

In conclusion, we show that the presence of underlying currents and infinite depth makes no difference on the position of extrema of dynamic pressure, which attains its maximum and minimum at the wave crest and wave trough respectively.

The remainder of this paper is organized as follows. In Section 2, we investigate the extrema of the dynamic pressure in a Stokes wave with uniform underlying currents and prove that the maximum and minimum of the dynamic pressure occur at the wave crest and the wave trough respectively, unaffected due to the presence of uniform underlying currents. Section 3 concerns the extrema of the dynamic pressure in deep-water Stokes wave flow.

2. The dynamic pressure of a stokes wave with underlying currents

2.1. The governing equations

The problems we consider are two-dimensional steady periodic irrotational gravity water waves over a homogeneous fluid with an underlying current strength k, the x-axis being the direction of wave propagation