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Global existence for nonlinear elastic waves in high space dimensions

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ABSTRACT

In Georgiev et al. (2005), the authors proved the global existence of solutions to the Cauchy problem of nonlinear elastic waves with memory for space dimensions $n \geq 3$. In this manuscript, we show that in high space dimensions $n \geq 4$, even without such memory effect, global solutions can be also constructed. The main tool we used is the vector fields method adapted to elastic waves in Sideris's work (Sideris, 2000).

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1. Introduction

This manuscript is devoted to study the following Cauchy problem for homogeneous, isotropic, hyper-elastic wave equations:

$$\begin{cases} \partial_t^2 u - c_2^2 \Delta u - (c_1^2 - c_2^2)\nabla\nabla\cdot u = N(\nabla u, \nabla^2 u), \\ t = 0 : u = u_0, \quad u_t = u_1. \end{cases} \tag{1.1}$$

Here $u(t, x) = (u^1(t, x), \ldots, u^n(t, x))$ denotes the displacement vector from the reference configuration, $\nabla = (\partial_1, \ldots, \partial_n)$, and the material constants c_1 (pressure wave speed) and c_2 (shear wave speed) satisfying $0 < c_2 < c_1$. The nonlinear term $N(\nabla u, \nabla^2 u)$ is linear in $\nabla^2 u$ and will be described explicitly in the later.

In the 3-D case ($n = 3$), local smooth solution for (1.1) in general will develop singularities even for small enough initial data and it will almost globally exist (see John [4,5], Klainerman and Sideris [8] and Tahvildar-Zadeh [12]). In order to ensure the global existence of smooth solutions with small initial data, some structural condition on the nonlinearity which is called null condition is necessary. We refer the reader to Agemi [1] and Sideris [11] (see also previous result in Sideris [9]). On the other hand, by adding some

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memory terms in the equation, Georgiev, Rubino and Sampalmieri [2] also established the global existence of solutions for space dimensions $n \geq 3$.

Our main goal in this manuscript is to establish the global existence of classical solutions to the Cauchy problem (1.1) in high space dimensions $n \geq 4$. We show that in high space dimensions $n \geq 4$, even without the memory effect in [2], global solutions can be also constructed. For quasilinear wave equations, such global existence results are first established by Klainerman [6,7] (see also the multiple speeds case in Hidano [3]). To achieve this goal, we will use the vector fields method adapted to elastic waves in Sideris [11].

An outline of this manuscript is as follows. The remainder of this introduction will be devoted to the description of some notations which will be used in the sequel, and a statement of the main global existence theorem. In Section 2, some necessary tools used to prove the global existence theorem are introduced, such as some properties of the commutation, Sobolev inequalities and the weighted decay estimates. Finally, the proof of the main theorem will be given in Section 3.

1.1. Notation

Denote the space–time gradient by

$$\partial = (\partial_t, \nabla) = (\partial_t, \partial_1, \partial_2, \ldots, \partial_n). \tag{1.2}$$

The angular momentum operators (generator of the spatial rotation) are the vector fields

$$\Omega = (\Omega_{ij} : 1 \leq i < j \leq n), \tag{1.3}$$

where

$$\Omega_{ij} = (x \wedge \nabla)_{ij} = x_i \partial_j - x_j \partial_i. \tag{1.4}$$

Denote the generators of simultaneous rotations by $\widetilde{\Omega} = (\widetilde{\Omega}_{ij} : 1 \leq i < j \leq n)$, where

$$\widetilde{\Omega}_{ij} = \Omega_{ij} I + U_{ij}, \tag{1.5}$$

$$(U_{ij})_{kl} = \begin{cases} 1, & (k,l) = (i,j); \\ -1, & (k,l) = (j,i); \\ 0, & \text{else case.} \end{cases} \tag{1.6}$$

We also use the scaling operator $\widetilde{S}$, where

$$\widetilde{S} = S - 1, \quad S = t\partial_t + r\partial_r. \tag{1.7}$$

All the above vector fields will be written as $\Gamma = (\partial, \widetilde{\Omega}, \widetilde{S})$.

Denote the linear elastic wave operator

$$L = \partial_t^2 - c_2^2 \Delta - (c_1^2 - c_2^2) \nabla \otimes \nabla. \tag{1.8}$$

The energy associated to the linear elastic wave operator L defined in (1.8) is

$$E_1(u(t)) = \frac{1}{2} \int_{\mathbb{R}^n} |\partial_t u(t,x)|^2 + c_2^2 |\nabla u(t,x)|^2 + (c_1^2 - c_2^2)(\nabla \cdot u(t,x))^2 dx, \tag{1.9}$$

and the higher order energies are

$$E_k(u(t)) = \sum_{|a| \leq k-1} E_1(\Gamma^a u(t)). \tag{1.10}$$

Denote the orthogonal projections onto radial and transverse directions by

$$P_1 u(x) = \frac{x}{r} \left(\frac{x}{r} \cdot u(x) \right) = \left(\frac{x}{r} \otimes \frac{x}{r} \right) u(x) \tag{1.11}$$

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