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On existence, regularity and uniqueness of thermally coupled incompressible flows in a system of three dimensional pipes

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ABSTRACT

We study an initial–boundary-value problem for time-dependent flows of heat-conducting viscous incompressible fluids in a system of three-dimensional pipes on a time interval $(0,T)$. We are motivated by the bounded domain approach with "do-nothing" boundary conditions. In terms of the velocity, pressure and temperature of the fluid, such flows are described by a coupled parabolic system with strong nonlinearities and including the natural boundary conditions for the velocity and temperature of the fluid on the part of the boundary where the fluid is supposed to leave the channel. The present analysis is devoted to the proof of the existence, regularity and uniqueness of the solution for the problem described above.

1. Introduction

Many problems of fluid thermo-mechanics involving unbounded domains occur in many areas of applications, e.g. flows of a liquid in duct systems, fluid flows through a thin or long pipe or through a system of pipes in hemodynamics, etc. From a numerical point of view, these formulations are not convenient and quite practical. Therefore, an efficient natural way is to cut off unbounded parts of the domain by introducing an artificial boundary in order to limit the computational work. Then the original problem posed in an unbounded domain is approximated by a problem in a smaller bounded computational region with appropriate boundary conditions prescribed at the cut boundaries. Hence, let Ω be a bounded domain in $\mathbb{R}^3$ with boundary $\partial\Omega$. In a physical sense, Ω represents a "truncated" region of an unbounded system

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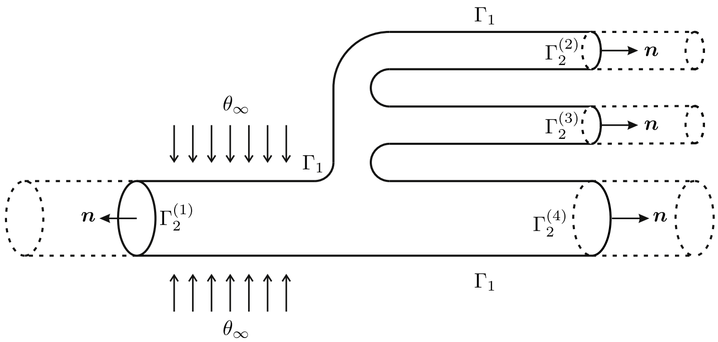

Fig. 1. Truncated piping system.

of pipes occupied by a moving heat-conducting viscous incompressible fluid. Γ_1 will denote the "lateral" surface and Γ_2 represents the open parts (cut boundaries) of the piping system. It is physically reasonable to assume that in/outflow pipe segments extend as straight pipes. More precisely, Γ_1 and Γ_2 are C^∞-smooth open disjoint not necessarily connected subsets of $\partial\Omega$ such that $\Gamma_2 = \bigcup_{i\in\mathcal{J}} \Gamma_2^{(i)}$, $\Gamma_2^{(i)} \cap \Gamma_2^{(j)} = \emptyset$ for $i \neq j$, $\partial\Omega = \overline{\Gamma}_1 \cup \overline{\Gamma}_2$, $\Gamma_1 \neq \emptyset$, $\Gamma_2 \neq \emptyset$, $\mathcal{M} = \partial\Omega - (\Gamma_1 \cup \Gamma_2) = \overline{\Gamma}_1 \cap \overline{\Gamma}_2 = \bigcup_{i\in\mathcal{J}} \mathcal{M}_i$, $\mathcal{J} = \{1,\ldots,d\}$, and the 2-dimensional measure of $\mathcal{M}$ is zero and $\mathcal{M}_i$ are smooth nonintersecting curves (this means that $\mathcal{M}_i$ are smooth curved nonintersecting edges and vertices (conical points) on $\partial\Omega$ are excluded). Moreover, all portions of Γ_2 are taken to be flat and Γ_1 and Γ_2 form a right angle $\omega_{\mathcal{M}} = \pi/2$ at all points of $\mathcal{M}$ (in the sense of tangential planes), see Fig. 1.

The flow of a viscous incompressible heat-conducting fluid is governed by balance equations for the linear momentum, mass and internal energy [19]:

$$\rho_0 \left(\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right) - \nu\Delta\boldsymbol{u} + \nabla P = \rho(\theta)\boldsymbol{f}, \tag{1.1}$$

$$\nabla\cdot(\rho_0\boldsymbol{u}) = 0, \tag{1.2}$$

$$c_v\rho(\theta)\left(\partial_t\theta + \boldsymbol{u}\cdot\nabla\theta\right) - \lambda\Delta\theta - \alpha_1\nu\mathbb{D}(\boldsymbol{u}) : \mathbb{D}(\boldsymbol{u}) = \alpha_2\rho(\theta)\boldsymbol{f}\cdot\boldsymbol{u} + h. \tag{1.3}$$

Here $\boldsymbol{u} = (u_1, u_2, u_3)$, P and θ denote the unknown velocity, pressure and temperature, respectively. The tensor $\mathbb{D}(\boldsymbol{u})$ denotes the symmetric part of the velocity gradient. The data of the problem are as follows: $\boldsymbol{f}$ is a body force and h a heat source term. Positive constant material coefficients represent the kinematic viscosity ν, reference density ρ_0, heat conductivity λ and specific heat at constant volume c_v. Following the well-known Boussinesq approximation, the temperature dependent density is used in the energy equation (1.3) and for computing the buoyancy force $\rho(\theta)\boldsymbol{f}$ on the right-hand side of Eq. (1.1). Everywhere else in the model, ρ is replaced by the reference value ρ_0. Throughout the paper, change of density ρ with temperature is given by strictly positive, nonincreasing and continuous function, such that

$$0 < \rho_1 \le \rho(\xi) \le \rho_2 < +\infty \quad \forall\xi \in \mathbb{R}\ (\rho_1, \rho_2 = \text{const}). \tag{1.4}$$

The energy balance equation (1.3) takes into account the phenomena of the viscous energy dissipation and adiabatic heat effects. For rigorous derivation of the model like (1.1)–(1.3) we refer the readers to [29], see also [19] for a brief review of theoretical issues concerning the Boussinesq approximation for thermally coupled time dependent flows. Rigorously derived asymptotic models describing stationary motion of heat-conducting incompressible viscous fluid through pipe-like domains can be found in [40,41].

To complete the model, suitable boundary and initial conditions have to be added. Concerning the boundary conditions of the flow, it is a standard situation to prescribe a homogeneous no-slip boundary condition for the velocity of the fluid on the fixed walls of the channel, i.e.

$$\boldsymbol{u} = \boldsymbol{0} \quad \text{on } \Gamma_1.$$