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Local well-posedness of strong solutions to density-dependent liquid crystal system

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A B S T R A C T

In this paper, we study the Cauchy problem to the density-dependent liquid crystal system in $\mathbb{R}^3$. We establish the local existence and uniqueness of strong solutions to this system. In order to overcome the difficulties caused by the high order coupling terms, a biharmonic regularization of the system, as an auxiliary system, is introduced, and we make full use of the intrinsic cancellation properties between the high order coupling terms.

1. Introduction

In this paper, we study the following density-dependent liquid crystal system on $\mathbb{R}^3$:

$$\begin{cases} \partial_t\rho + u\cdot\nabla\rho = 0, \\ \rho(\partial_t u + (u\cdot\nabla)u) + \nabla P = \Delta u - \nabla\cdot[\nabla d\odot\nabla d + (\Delta d + |\nabla d|^2 d)\otimes d], \\ \operatorname{div} u = 0, \\ \partial_t d + (u\cdot\nabla)d - (d\cdot\nabla)u = (\Delta d + |\nabla d|^2 d) - (d^T A d)d, \\ |d| = 1, \end{cases} \tag{1.1}$$

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where $\rho \in [0, \infty)$ is the density, $u = (u^1, u^2, u^3)$ is the velocity field, $P \in \mathbb{R}$ is the pressure, and $d = (d^1, d^2, d^3) \in S^2$, the unit sphere in $\mathbb{R}^3$, is the director field, $A = \frac{1}{2}(\nabla u + (\nabla u)^T)$ and $\nabla d \odot \nabla d = (\partial_i d \cdot \partial_j d)_{3\times 3}$.

For the homogeneous case, i.e. the case that the density is a constant, the mathematical studies on the dynamical liquid crystal systems were started by Lin–Liu [24,25], where they established the global existence of weak solutions, in both 2D and 3D, to the Ginzburg–Landau approximation of the liquid crystal system, see Cavaterra–Rocca–Wu [1] and Sun–Liu [30] for some generalizations to the general liquid crystal systems, but still with Ginzburg–Landau approximation. Global existence of weak solutions to the original liquid crystal systems in 2D, without the Ginzburg–Landau approximation, was established by Lin–Lin–Wang [23], Hong [8], Hong–Xin [10], Huang–Lin–Wang [12] and Wang–Wang [31], and in particular, it was shown that global weak solutions to liquid crystal system in 2D have at most finite many singular times, while the uniqueness of weak solutions to liquid crystal system in 2D was proved by Lin–Wang [26], Li–Titi–Xin [18], Wang–Wang–Zhang [32] and Xu–Zhang [37]; global existence (but without uniqueness) of weak solutions to the liquid crystal system in 3D was recently established by Lin–Wang [27], under the assumption that the initial director filed d_0 takes value from the upper half unit sphere. If the initial data are suitably smooth, then the liquid crystal system has a unique local strong solution, see Hong–Li–Xin [9], Li–Xin [19], Wang–Wang [31], Wang–Zhang–Zhang [33], Wu–Xu–Liu [35] and Hieber–Nesensohn–Prüs–Schade [6]. Moreover, if the initial data is suitably small, or the initial director filed satisfies some geometrical condition in 2D, then the local strong solution to the liquid crystal system can be extended to be a global one, see Hu–Wang [11], Lei–Li–Zhang [15], Li–Wang [21], Ma–Gong–Li [29], Gong–Huang–Liu–Liu [5] and Hieber–Nesensohn–Prüs–Schade [6]; remarkably, the recent work of Huang–Lin–Liu–Wang [13], concerning the finite time blow up of liquid crystal system in 3D, indicates that one cannot generally expect the global existence of strong solutions to the liquid crystal system in 3D, without any further assumptions, beyond the necessary regularities, on the initial data. It is worth to mention that some mathematical analysis concerning the global existence of weak solutions and local or global well-posedness of strong solutions of the non-isothermal liquid crystal systems were addressed by Hieber–Prüs [7], Li–Xin [20], Feireisl–Rocca–Schimperna [4] and Feireisl–Frémond–Rocca–Schimperna [3].

In contrast with the homogeneous case, there are much less works concerning the mathematical analysis on the inhomogeneous liquid crystal system. As the counterparts of [24], global existence of weak solutions to the Ginzburg–Landau approximated density-dependent liquid crystal system was established by Liu–Zhang [28], Xu–Tan [36] and Jiang–Tan [14], while for the original liquid crystal system, i.e. the system without the Ginzburg–Landau approximation, global existence of weak solutions was established only in 2D and under some geometrical assumptions on the initial director field, see Li [16], where the global existence of strong solutions was also established at the same time. Local well-posedness of strong solutions to the three dimensional liquid crystal system was established by Wen–Ding [34], while the corresponding global well-posedness, under some smallness assumption on the initial data, was proved by Li–Wang [22] in the absence of vacuum, and by Li [17] and Ding–Huang–Xia [2] in the presence of vacuum.

Noticing that in all the existing works concerning the density-dependent liquid crystal systems, see, e.g., [28,36,14,16,34,22,17,2] mentioned in the above paragraph, the systems under consideration are simplified versions of the general liquid crystal system. In the general liquid crystal system, there presents some high order coupling terms which are as high as the leading terms, while these coupling terms were ignored in the works just mentioned. Technically, the main difficulty resulted from the high order coupling terms is: on the one hand, in order to get the energy estimates, one has to make full use of the intrinsic cancellation properties between these high order coupling terms, but on the other hand, if we solve the system by using the standard arguments such as the Galerkin approximation or linearization, it will destroy these intrinsic cancellation properties.

The aim of this paper is to study the density-dependent liquid crystal system, and we focus on such a system that has the high order coupling terms. Precisely, we consider system (1.1), and study the Cauchy