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Irrotational two-dimensional free-surface steady water flows over a flat bed with underlying currents

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ABSTRACT

We investigate the free boundary non-linear problem for large amplitude smooth Stokes waves in flows of finite depth with underlying currents. The qualitative nature of the flow and pressure fields is investigated using maximum principles. Bounds on the surface profile have been derived. Influence of the relative magnitude of wave and current speed on the flow and pressure fields has been shown. The possible co-existence of propagating surface waves and an underlying uniform current at the wave speed is settled—this can only happen if the surface is flat.

1. Introduction

The topic of Stokes waves of large amplitude is an interesting hydrodynamic problem which has drawn attention of researchers in the recent years. Contributions to analytical theories and studies related to deep-water waves mainly have been due to the work carried out by Toland and his co-workers (e.g., [2,1,23,24,4]). An irrotational two-dimensional periodic wave solution of the governing fluid equations of motion with a free surface flowing over a flat bed, and possessing (i) a symmetric profile, (ii) a crest and a trough per wavelength and (iii) a constant speed at the surface, leads to a Stokes wave over a flat bed. The only driving force of action in this case is the gravity. Such a problem can be mathematically formulated as a nonlinear wave propagation problem with free boundary for harmonic functions over a planar domain and exhibits rich mathematical structures. This allows the study of the Stokes wave problem of large amplitude following mathematical approaches like the maximum principles. Strong qualitative conclusions can be derived from maximum principles for harmonic functions in combination with exploiting some of the physical structures of the problem itself. Significant amount of research has been carried out by Constantin and his colleagues on investigating the structures and behavior of Stokes wave, pressure and flow beneath the surface and

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trajectories of particles [12,14,16,9,7,15,6]. Some other noteworthy work has been carried out in this area of nonlinear water waves (see [20,19,21,5,25,8]). However, large amplitude Stokes waves in flows of finite depth with an underlying current have received little attention so far.

The nonlinear interaction between current and wave in flows over finite depth influencing the ocean dynamics and sediment transportation is not unknown [18]. Several regions around the world witness the effect of surface waves and internal current. The Bay of Biscay is one of the world's strongest generation sites for internal current where current amplitudes of 25 cm/s have been observed [17]. A maximum value of 12 cm/s of current has been observed at a station over the French continental slope down to a water depth of 2300 m [22]. This also validates the influence of internal current propagating on the flow field as measurements confirm that the barotropic tidal current due to the surface tide is only 3 cm/s. Measurements at a point further offshore located at the south edge of the Merriadzeck Terrace revealed an increase in the velocity of the current near the deepest level at 2400 m reaching a value more than 20 cm/s [22]. Scottish coasts are mainly influenced by North Atlantic currents. In addition, a jet-like current along the edge of the continental slope with speeds in the range of 15–30 cm/s exists centered approximately over a 400–500 m depth contour [3]. Tidal currents are intensified in localized areas, usually when the topography constrains the flow such as between Orkney and Shetland, where tidal currents can be as high as 3.5–4.5 m/s [3]. Speed of current is observed to be relatively slower in Galway bay test site in the west of Ireland on a typical summer day. Real time observations are provided by Marine Institute at the Spiddal wave buoy site (available from www.oceanenergyireland.com) and the mean current speed on a typical day of June 2016 has been observed to be 0.26 m/s at a location 1.5 km offshore with a shallow water depth less than 25 m.

This paper aims to study the qualitative behavior of large amplitude smooth Stokes waves in flows of finite depth with uniform current. Velocity fields, pressure fields and surface profiles for flow under the joint influence of surface waves and an underlying uniform current are investigated and results on their qualitative nature have been proved. Since the explicit functional form of the surface profile is unknown, the current state-of-art on flow patterns and underneath pressure for large amplitude Stokes waves with underlying current is still not fully solved. Only recently some information and solutions are available from studies on nonlinear large amplitude Stokes wave over a finite depth [7,15]. In this paper, maximum principles for harmonic functions on elliptic equations in combination with some structural properties of physical boundary conditions provide a way to unveil some of the structures of the flow and pressure fields as well as surface profile of the free boundary. Three specific situations arise depending on the relative speed of the surface waves and the average strength (speed) of the underlying current. It is shown that when the speed of the current is greater than the wave speed, the nature of the flow fields is different from what is expected where no underlying current was considered [7,10]. The pressure field however is unaffected due to the presence of underlying currents. Further, we prove that a Stokes surface wave and an underlying uniform current of same speed cannot coexist in an irrotational two dimensional flow field.

This paper is organized as follows. In Section 2 we present the governing equations of motion. Section 3 establishes the framework for flow with currents. Section 4 analyses the case when the speed of the current exceeds the speed of surface waves. Qualitative nature of flow and pressure fields is obtained and proved. Bounds on surface profiles have also been presented. Sections 5 and 6 analyze the cases when the speed of the current is equal to and less than the surface wave velocity respectively followed by some comments in Section 7.

1.1. Problem definition

We seek an irrotational smooth solution to the governing equations of fluid dynamics represented by the Euler equations for which there exist a period $L > 0$, a wave speed c, and a current strength K such that