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Regional control in optimal harvesting of population dynamics

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ABSTRACT

Here we investigate the regional control for some optimal harvesting problems related to population dynamics; namely we consider the problem of maximizing the profit for spatially structured harvesting problems with respect to both the harvesting effort and the selected subregion ω (of the whole domain Ω) where the effort is localized. For a fixed subregion ω we state necessary optimality conditions and use them to get the structure of the optimal effort and to reformulate the maximization problem with respect to the subregion ω, where the harvesting effort is localized, in a more convenient way. We derive an iterative algorithm to increase at each iteration the profit by changing the subregion where the effort is localized. Some numerical tests are given to illustrate the effectiveness of the results for a particular optimal harvesting problem. Final comments are given as well concerning further directions to extend the results and methods presented here.

1. Introduction and setting of the problem

Optimal harvesting problems have been intensively studied for structured population dynamics by several authors. We wish further to evidence that possible space (and time) heterogeneities of the domain Ω are taken in due account via the parameters of the relevant dynamical system. The main attention has been paid in deriving first order necessary optimality conditions and to use gradient-type algorithms to approximate an optimal effort (control).

Spatially structured harvesting problems have usually been treated by considering an effort applied to the whole domain Ω of interest for the relevant population (see for example [2]). Actually it should be obvious that, on the basis of suitable information, the effort is localized in a subregion ω of Ω (see [5]).

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Hence the point we wish to raise in this paper regards not only the optimization with respect to the intensity of the harvesting effort, but also the optimal choice of the subregion ω where it is implemented.

We wish to evidence that such problems may be treated as shape optimization problems. Hence, to begin with, we remind that from a measure theoretic point of view the geometry of a set ω can be characterized in terms of its Minkowski functionals, or intrinsic volumes. In 2D there are three such functionals and these are proportional to more commonly known quantities such as the area, the perimeter and the Euler–Poincaré characteristic; see [26], [31, p. 30]. Intrinsic volumes have been successfully used in material science to characterize and discriminate morphology of various media; see [38,7].

In this paper we shall limit ourselves to refer to two of such functionals, the area, and the perimeter. On the other hand it is of course easily understandable, from the point of view of our optimal control problems that, in order to optimize the harvesting region, we have to act on both the area and the perimeter.

A convenient tool for handling the shape of a domain ω in $\mathbb{R}^2$ is the so called implicit interface representation, according to which the boundary of a domain is defined as the isocontour of some function φ (see [37, p. 3], [19, p. 75]). Assume that in the domain $\Omega \subset \mathbb{R}^2$ we may represent a subset ω by means of a function $\varphi : \Omega \to \mathbb{R}$ called its implicit function as $\omega = \{x \in \Omega;\ \varphi(x) \geq 0\}$ (or $\omega = \{x \in \Omega;\ \varphi(x) \leq 0\}$), while its boundary is defined as the zero level set of φ: $\partial\omega = \{x \in \Omega;\ \varphi(x) = 0\}$. In this way the interior of ω has the following implicit representation $Int(\omega) = \{x \in \Omega; \varphi(x) > 0\}$.

A useful tool to carry out integration over the interior of the domain ω is then the Heaviside function $H : \mathbb{R} \to \{0, 1\}$, such that

$$H(z) = \begin{cases} 1, & \text{if } z \geq 0 \\ 0, & \text{if } z < 0. \end{cases}$$

If φ is the implicit function of ω, in order to integrate over ω a function f defined over the whole Ω we may write $\int_\Omega f(x)H(\varphi(x))dx$. If we need to integrate over the boundary of ω we may then refer to the Dirac Delta function as the generalized derivative of the Heaviside function H. In fact, if φ is sufficiently smooth, the directional derivative of the Heaviside function in the normal direction at a point $x \in \partial\omega$ is given by $\widehat{\delta}(x) = H'(\varphi(x))|\nabla\varphi(x)|$, and, by using the usual Dirac Delta δ on $\mathbb{R}$, we have $\widehat{\delta}(x) = \delta(\varphi(x))|\nabla\varphi(x)|$. Hence the boundary integral over $\partial\omega$ of a function f defined over the whole Ω may be expressed as $\int_\Omega f(x)\delta(\varphi(x))|\nabla\varphi(x)|dx$. In the above it has been taken into account that, in terms of the implicit function φ, the unit (outward) normal to the boundary at point $x \in \partial\omega$ can be expressed as $n(x) = \frac{\nabla\varphi(x)}{|\nabla\varphi(x)|}$.

We wish to mention that one might rephrase the above control problem in terms of suitable measures, as done in [12,17,18]; indeed they have treated the problem of existence for a generic optimal harvesting effort, with constraints. Our approach has allowed us to take advantage of the meaning and of the specific structure of our system.

We denote by $y^u(x,t)$ the population density at position x and time t of a population that is free to move in a habitat $\Omega \subset \mathbb{R}^2$ and is subject to a harvesting effort $u(x,t)$ (localized in a subregion ω, with $t \in [0,T]$). Our main purpose is to investigate the following problem:

$$\underset{\omega}{Maximize}\ \underset{u\in\mathcal{U}_\omega}{Maximize}\ \ \Phi_\omega(u), \qquad \textbf{(OH)}$$

where $\Phi_\omega(u) = \int_0^T \int_\omega [u(x,t)y^u(x,t) - cu(x,t)]dx\ dt - c_1\text{length}(\partial\omega) - c_2\text{area}(\omega)$, and $\mathcal{U}_\omega = \{v \in L^\infty(\omega \times (0,T));\ 0 \leq v(x,t) \leq L \text{ a.e. in } \omega \times (0,T)\}$, T, L, c_1, c_2 are positive constants and $c \in [0, +\infty)$ (c represents the cost per unit of the harvesting effort, $\int_0^T \int_\omega cu(x,t)dx\ dt$ is the cost of the total harvesting effort, and $c_1\text{length}(\partial\omega) + c_2\text{area}(\omega)$ is the cost paid to harvest in the specific subregion ω). It means that we wish to maximize (with respect to the subregion ω and to the harvesting effort) the profit, which is the difference between the gain of the harvested population and the cost of the harvesting effort plus the cost paid to harvest in the subregion ω.