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Higher integrability for minimizers of asymptotically convex integrals with discontinuous coefficients

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This paper is dedicated to Nicola Fusco, who directed us to the theory of regularity in the Calculus of Variations, on the occasion of his 60th birthday

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ABSTRACT

We study the local regularity of vectorial minimizers of integral functionals with standard p-growth. We assume that the non-homogeneous densities are uniformly convex and have a radial structure, with respect to the gradient variable, only at infinity. Under a $W^{1,n}$-Sobolev dependence on the spatial variable of the integrand, n being the space dimension, we show that the minimizers have the gradient locally in L^q for every $q > p$. As a consequence, they are locally α-Hölder continuous for every $\alpha < 1$.

1. Introduction

In this paper we study the regularity of vectorial local minimizers of functionals with irregular integrands in the x-variable and only asymptotically convex with respect to the gradient variable. In order to state our

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result precisely, we introduce right now our hypotheses. We will consider

$$\mathcal{F}(u;\Omega) := \int_\Omega f(x, Du)\,dx, \tag{1.1}$$

where $\Omega \subset \mathbb{R}^n$, $n > 2$, is a bounded open set, $u : \Omega \to \mathbb{R}^N$, $N > 1$, is a Sobolev map and $f : \Omega \times \mathbb{R}^{nN} \to [0,+\infty)$ is a Carathéodory function convex with respect to the second variable.

As it is well known since the famous example by De Giorgi [13] (see also [39,40,45]), in order to avoid the irregularity phenomena peculiar of the vectorial minimizers, the dependence of the energy density on the modulus of the gradient variable is necessary. We shall assume it only *at infinity*, i.e., for large values of the gradient variable ξ. Precisely:

(A1) there exist $\tilde{R} > 0$ and a function $\tilde{f} : \Omega \times [\tilde{R},+\infty) \to [0,+\infty)$ such that

$$f(x,\xi) = \tilde{f}(x,|\xi|), \tag{1.2}$$

for a.e. $x \in \Omega$ and every $\xi \in \mathbb{R}^{nN} \setminus B_{\tilde{R}}(0)$.

The integrand f will satisfy the so-called *p-growth condition*, that is

(A2) there exist an exponent $p > 1$ and constants $c_1, c_2, L > 0$ such that

$$c_1|\xi|^p - c_2 \le f(x,\xi) \le L(1+|\xi|)^p,$$

for a.e. $x \in \Omega$ and $\xi \in \mathbb{R}^{nN}$.

The usual *p-uniform convexity* will be assumed only *at infinity*. More precisely, we shall suppose that $\xi \to f(x,\xi) \in C^2(\mathbb{R}^{nN} \setminus B_{\tilde{R}}(0))$ and

(A3) there exists $\nu > 0$ such that

$$\langle D_{\xi\xi} f(x,\xi)\lambda, \lambda\rangle \ge \nu\,(1+|\xi|)^{p-2}|\lambda|^2,$$

for a.e. $x \in \Omega$, for every $\xi \in \mathbb{R}^{nN} \setminus B_{\tilde{R}}(0)$ and for every $\lambda \in \mathbb{R}^{nN}$.

Note that, since f is C^2 with respect to the gradient variable outside the ball $B_{\tilde{R}}(0)$, the assumption in (A3) is equivalent to the C^2- asymptotic convexity introduced in [5].

Also the bound on the second order derivatives in the gradient variable will be required only at infinity. Indeed, we shall assume that

(A4) there exists $L_1 > 0$ such that

$$|D_{\xi\xi} f(x,\xi)| \le L_1(1+|\xi|)^{p-2},$$

for a.e. $x \in \Omega$ and every $\xi \in \mathbb{R}^{nN} \setminus B_{\tilde{R}}(0)$.

We now introduce the main property of our energy density. As already mentioned, we will not ask a regular dependence of f on the x-variable. Indeed, the function $x \to D_\xi f(x,\xi)$ will be required to be weakly differentiable for every $\xi \in \mathbb{R}^{nN} \setminus B_{\tilde{R}}(0)$ and it will be assumed that

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