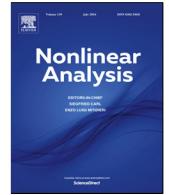




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Quasilinear elliptic systems with measure data

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ABSTRACT

We study the existence of solutions of quasilinear elliptic systems involving N equations and a measure on the right hand side, with the form

$$\begin{cases} -\sum_{i=1}^n \frac{\partial}{\partial x_i} \left(\sum_{\beta=1}^N \sum_{j=1}^n a_{i,j}^{\alpha,\beta}(x,u) \frac{\partial}{\partial x_j} u^\beta \right) = \mu^\alpha & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\alpha \in \{1, \dots, N\}$ is the equation index, Ω is an open bounded subset of \mathbb{R}^n , $u : \Omega \rightarrow \mathbb{R}^N$ and μ is a finite Radon measure on \mathbb{R}^n with values in \mathbb{R}^N . Existence of a solution is proved for two different sets of assumptions on A . Examples are provided that satisfy our conditions, but do not satisfy conditions required on previous works on this matter.

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1. Introduction

Let us consider the Dirichlet elliptic problem

$$-\operatorname{div} [A(x, u(x), Du(x))] = \mu \quad \text{in } \Omega, \quad (1.1)$$

$$u = 0 \quad \text{on } \partial\Omega, \quad (1.2)$$

where $u : \Omega \subset \mathbb{R}^n \rightarrow \mathbb{R}^N$, μ is a measure on \mathbb{R}^n with values into \mathbb{R}^N and A satisfies suitable coercivity and growth conditions. We note that (1.1) is a system of N equations.

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First consider the case $N = 1$, i.e. (1.1) is only one single equation. Existence of distributional solutions $u : \Omega \subset \mathbb{R}^n \rightarrow \mathbb{R}$ has been deeply studied, starting from [7], see [9,11,8,32,4] and the survey [5]. Uniqueness seems to be a delicate matter, e.g. see [33,3,19] and the introduction of [12]. Regularity results are contained in [28–30,10,22,2] and the survey [31] (see also [6]). Note that existence of solutions is usually obtained by a truncation argument, which shows why the vectorial case $N \geq 2$ is difficult and only few contributions are available in the literature. In fact, for systems $N \geq 2$, the p -Laplacian $A(x, y, \xi) = |\xi|^{p-2}\xi$ is treated in [18,13], and the anisotropic case, in which each component of the gradient $D_i u$ may have a possibly different exponent p_i , is dealt in [23,24]. Let us write (1.1) using components, that is,

$$-\sum_{i=1}^n \frac{\partial}{\partial x_i} [A_i^\alpha(x, u(x), Du(x))] = \mu^\alpha \quad \text{for } \alpha \in \{1, \dots, N\}. \tag{1.3}$$

We note that systems more general than the p -Laplacian are considered in [14,16], under the assumption

$$0 \leq \sum_{\alpha=1}^N \sum_{i=1}^n A_i^\alpha(x, y, \xi) ((Id - b \times b)\xi)_i^\alpha \tag{1.4}$$

for every $b \in \mathbb{R}^N$ with $|b| \leq 1$. In [34], the author assumes the componentwise sign condition

$$0 \leq \sum_{i=1}^n A_i^\alpha(x, y, \xi) \xi_i^\alpha \tag{1.5}$$

for every $\alpha \in \{1, \dots, N\}$. When $N = 2$, (1.4) implies (1.5), since it is enough to take first $b = (1, 0)$ and then $b = (0, 1)$. In [25], the authors consider that A is independent of y and satisfies the componentwise coercivity condition

$$\nu |\xi^\alpha|^2 - M \leq \sum_{i=1}^n A_i^\alpha(x, \xi) \xi_i^\alpha \tag{1.6}$$

for every $\alpha \in \{1, \dots, N\}$, for some constants $\nu \in (0, +\infty)$ and $M \in [0, +\infty)$. In [15], they relax (1.4) to some extent

$$-c|\xi|^q - g(x) \leq \sum_{\alpha=1}^N \sum_{i=1}^n A_i^\alpha(x, y, \xi) ((Id - b \times b)\xi)_i^\alpha \tag{1.7}$$

for some $c \in [0, +\infty)$, $g \in L^1(\Omega)$ and $q \in [1, n)$ where $\xi \mapsto A(x, y, \xi)$ is n -coercive.

We note that in [19,17,26] the authors do not truncate u ; they modify Du and then adjust via Hodge decomposition; such a procedure requires the dimension n to be the exponent in the coercivity condition for A . The authors use nice estimates for Hodge decomposition, which have been studied in [20] (see also appendix A, in [21]).

In the present paper we consider quasilinear systems, i.e. systems (1.3) with

$$A_i^\alpha(x, y, \xi) = \sum_{\beta=1}^N \sum_{j=1}^n a_{i,j}^{\alpha,\beta}(x, y) \xi_j^\beta, \tag{QL}$$

where the coefficients $a_{i,j}^{\alpha,\beta}(x, y)$ are measurable with respect to x and continuous with respect to y . Moreover, we assume ellipticity for the diagonal coefficients $a_{i,j}^{\alpha,\alpha}$, the off-diagonal coefficients $a_{i,j}^{\alpha,\beta}$ (with $\alpha \neq \beta$) are sufficiently small, and all the coefficients are bounded. We prove existence of distributional solutions to (1.1)–(1.2) under two sets of hypotheses, with different assumptions on the off-diagonal coefficients.

The first result deals with off-diagonal coefficients $a_{i,j}^{\alpha,\beta}$ (with $\alpha \neq \beta$) that have support contained in a “staircase” set, along the diagonals of the $y^\alpha - y^\beta$ plane (see assumption (A_5) in Section 2 and Fig. 1).

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