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Quantitative asymptotic estimates for evolution problems

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We study a quantitative asymptotic, stability estimates for solutions to nonlinear evolution equations. More precisely, we measure the distance in time of a solution to a parabolic problem from a solution to a stationary one.

"*To Nicola, nothing else to say. He knows*"

1. Introduction

In this paper we consider nonlinear degenerate parabolic equations whose model is the p-harmonic evolution equation. Our goal is to estimate the distance in time of a solution to such evolution problem from a solution of a stationary p-Laplace type equation.

Although the problems are nonlinear, we will see that under small perturbations of the data the distance between the solutions of two problems at every time remains stable.

To describe our results we confine ourselves to the model case.

Let Ω be a bounded open set of $\mathbb{R}^N$, $N > 2$, and

$$A(x,t,\xi) = \langle \mathcal{A}(x,t)\xi, \xi\rangle^{\frac{p-2}{2}} \mathcal{A}(x,t)\xi \quad 2 \le p < N,$$

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where $\mathcal{A}(x,t)$ is a measurable, symmetric, matrix field defined on $\Omega_\infty = \Omega \times (0,+\infty)$ satisfying

$$\alpha|\xi|^2 \leq \langle \mathcal{A}(x,t)\xi, \xi\rangle \leq \beta|\xi|^2$$

for every $\xi \in \mathbb{R}^N$ and for almost every $(x,t) \in \Omega_\infty$, where $0 < \alpha \leq \beta$. We consider the p-harmonic evolution problem

$$\begin{cases} u_t - \text{div}\,(A(x,t,\nabla u)) = F & \text{in } \Omega_\infty \\ u = 0 & \text{on } \partial\Omega \times (0,+\infty) \\ u(x,0) = u_0(x) & \text{on } \Omega \end{cases} \tag{1.1}$$

where $u_0 \in L^2(\Omega)$ and $F \in L^{p'}(0,+\infty; W^{-1,p'}(\Omega))$, $pp' = p + p'$.

It is well known that the vector field $A : \Omega \times (0,+\infty) \times \mathbb{R}^N \to \mathbb{R}^N$ is a Carathéodory function which is strongly monotone. Hence, problem (1.1) admits a unique solution (see [14]).

There is a wide literature concerning decay estimates for solutions to homogeneous evolution problems (i.e. problems in divergence form as (1.1) with $F \equiv 0$) and in this case ultracontractive and supercontractive bounds on u are available (see for example [2–4,6,9,15–17,20–22] and the references therein).

When F is a Borel measure with finite total mass, pointwise estimates of the spatial gradient of solution to equation in (1.1) have been recently established in [11,12] in terms of Riesz and Wolff potential, respectively.

Here we give estimates on the L^2-distance of u from the solution w of the stationary problem

$$\begin{cases} -\text{div}\,(|\nabla w|^{p-2}\nabla w) = f & \text{in } \Omega \\ w = 0 & \text{on } \partial\Omega \end{cases} \tag{1.2}$$

with $f \in W^{-1,p'}(\Omega)$, in terms of the characteristic of the matrix $A = A(x,t)$, defined as the quantity

$$K(t) = \operatorname*{ess\,sup}_{x\in\Omega} (1 + |\mathcal{A}(x,t) - Id|)^{\frac{p}{2}}$$

for a.e. $t > 0$. Here Id denotes the identity matrix. The function $K(t)$ can be considered the distance in time between the operator $A(x,t,\xi)$ and the p-Laplacian operator [7,8,10]. More precisely, our result reads

Theorem 1.1. *Assume $p > 2$ and let $u \in C_{\rm loc}([0,+\infty); L^2(\Omega)) \cap L^p_{\rm loc}([0,+\infty); W^{1,p}_0(\Omega))$ and $w \in W^{1,p}_0(\Omega)$ be solutions of (1.1) and (1.2) respectively. Then there exist constants C_i, $i = 0,1$, such that the following estimate*

$$\|u(t) - w\|^2_{L^2(\Omega)} \leq \frac{\|u_0 - w\|^2_{L^2(\Omega)}}{\left[1 + C_1\|u_0 - w\|^{p-2}_{L^2(\Omega)} t\right]^{\frac{2}{p-2}}} + C_0 \int_0^t g(s)ds \tag{1.3}$$

holds for every $t > 0$. Here

$$g(t) = \left[(K(t) - 1)^{p'} \|\nabla w\|^p_{L^p(\Omega)} + \|F(t) - f\|^{p'}_{W^{-1,p'}(\Omega)}\right].$$

Moreover, there exist positive constants c and t_1 such that

$$\|u(t) - w\|^2_{L^2(\Omega)} \leq \frac{c}{t^{\frac{2}{p-2}}} + C_0 \int_{\frac{t}{2}}^t g(s)ds \tag{1.4}$$

holds for every $t \geq t_1$.

The constants $C_i = C_i(\alpha,p,N)$, $i = 0,1$, and $c = c(\alpha,p,N)$ are respectively given by (4.9), (4.13) and (4.14) where $\lambda = \frac{\alpha^{p/2}}{2}$.

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