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Embedded solitons with $\chi^{(2)}$ and $\chi^{(3)}$ nonlinear susceptibilities by extended trial equation method

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ABSTRACT

This paper employs the extended trial equations algorithm to extract soliton and other forms of waves in quadratic-cubic nonlinear medium. These waves stem from the continuous spectrum. The solutions appear with their corresponding constraints, also known as the existence criteria for the waves.

1. Introduction

There are several mathematical methods and tools to study various forms of nonlinear evolution equations that appear in optics and other areas of mathematical physics [1–10]. A few of these are mapping method, variational principle, Lie symmetry analysis, Kudryashov's method and several others. This paper will employ one such integration scheme, namely the extended trial function scheme to address a fairly new problem from nonlinear optics. It is the study of embedded solitons that appear with quadratic-cubic nonlinearity. These embedded solitons appear in the continuous regime of the spectrum and is therefore less visible in this area of research. The study of such solitons with quadratic-cubic nonlinearity first appeared during 2008 [7]. Later, this problem was further studied by the aid of Lie symmetry analysis and mapping methods to extract various kinds of solitons along with the conservation law [9]. This paper serves as a sequel to previously reported results and is therefore going to focus on the application of extended trial equation method to retrieve embedded solitons. The details are discussed in the subsequent sections.

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1.1. Governing equations

The governing equation for solitons in quadratic nonlinear media is given by [7,9]:

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + c_1 q^* r + d_1 |q|^2 q = 0, \tag{1}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + c_2 r + d_2 q^2 + \delta |q|^2 r = 0. \tag{2}$$

In Eqs. (1) and (2), the dependent variables are $q(x, t)$ and $r(x, t)$ which are complex valued functions representing fundamental harmonic (FH) and second harmonic (SH), respectively. The independent variables x and t are spatial and temporal variables, respectively. The coefficients a_j and b_j are from group velocity dispersion and spatio-temporal dispersion, respectively. Then, c_j stands for group-velocity mismatch because of frequency difference between FH and SH fields.

2. Mathematical analysis

The extended trial function approach [2–6,10] is one of the most popular modern approaches to obtain the solutions to such NLEEs since its first appearance a few years ago. In fact, this approach has been successfully implemented to extract soliton solutions to water wave model, plasma physics model as well as in nonlinear optics and nuclear physics. In order to study the details of this model, the hypothesis is [1,7,9]:

$$q(x, t) = P_1(\zeta) e^{i\phi(x,t)}, \tag{3}$$

$$r(x, t) = P_2(\zeta) e^{2i\phi(x,t)}, \tag{4}$$

where the wave variable ζ is given by

$$\zeta = x - vt. \tag{5}$$

Here, $P_l(\zeta)$ for $l = 1, 2$ represents the amplitude component and v is the speed of the wave, while phase is defined as

$$\phi(x, t) = -\kappa x + \omega t + \theta, \tag{6}$$

where κ is the soliton frequency, ω is the soliton wave number and θ the phase constant. Substituting (3) and (4) into (1) and (2) and decomposing into real and imaginary parts gives

$$(b_1 v - a_1) P_1'' + \left(\omega + a_1 \kappa^2 - b_1 \omega \kappa\right) P_1 - c_1 P_1 P_2 - d_1 P_1^3 = 0, \tag{7}$$

and

$$v = \frac{b_1 \omega - 2a_1 \kappa}{1 - b_1 \kappa}, \tag{8}$$

respectively, from the first component. Similarly, the second component respectively gives

$$(b_2 v - a_2) P_2'' + \left(2\omega + 4a_2 \kappa^2 - 4b_2 \omega \kappa - c_2\right) P_2 - d_2 P_1^2 - \delta P_1^2 P_2 = 0, \tag{9}$$

and

$$v = \frac{2b_2 \omega - 4a_2 \kappa}{1 - 2b_2 \kappa}. \tag{10}$$

From (8) and (10) equating the two values of the speed v leads to

$$a_1 = 2a_2, \tag{11}$$

and

$$b_1 = 2b_2. \tag{12}$$

Thus, the governing equations modify to

$$iq_t + 2aq_{xx} + 2bq_{xt} + c_1 q^* r + d_1 |q|^2 q = 0, \tag{13}$$

$$ir_t + ar_{xx} + br_{xt} + c_2 r + d_2 q^2 + \delta |q|^2 r = 0. \tag{14}$$

Hence, the real part equations from the two components can be written as

$$2(bv - a) P_1'' + \left(\omega + 2a\kappa^2 - 2b\omega\kappa\right) P_1 - c_1 P_1 P_2 - d_1 P_1^3 = 0, \tag{15}$$

and

$$(bv - a) P_2'' + \left(2\omega + 4a\kappa^2 - 4b\omega\kappa - c_2\right) P_2 - d_2 P_1^2 - \delta P_1^2 P_2 = 0. \tag{16}$$

These two equations will be further studied in the next subsection to retrieve solitons and other forms of wave solutions.