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Original research article

Scattering from a cylindrical obstacle deeply buried beneath a planar non-integer dimensional dielectric slab using Kobayashi potential method

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ABSTRACT

Mathematical formulation for the far-zone field scattered by a cylindrical obstacle deeply buried beneath a planar dielectric slab has been presented. Method known as Kobayashi potential method has been employed for this purpose and transverse electric uniform plane wave has been used as a source of excitation. It has been assumed that dimension of space within slab is non-integer in the direction normal to planar interfaces of the slab. Moreover, it has also been assumed that distance of each interface of the slab from obstacle is very large as compared to size of the obstacle. Due to this assumption scattered field after reflection from the slab has no interaction with the obstacle and the situation has been called deeply buried. Field pattern has also been obtained and compared with the published work.

1. Introduction

The knowledge of characterization of material over a wide frequency range, propagation, scattering and radiation of electromagnetic waves are the essential requirements for accurate design and modeling of several engineering problems [1–4]. For example: scattering of electromagnetic waves has applications in remote sensing [5], design of printed circuit boards [6] and electromagnetic shielding [7], etc. The behavior of materials may depend on various parameters: frequency of excitation, position and direction of excitation, etc. The dispersive materials can be modeled by frequency dependent complex relative permittivity and/or magnetic permeability as

$$\epsilon(\omega) = \epsilon'(\omega) - i\epsilon''(\omega)$$

$$\mu(\omega) = \mu'(\omega) - i\mu''(\omega)$$

where ω is the operating angular frequency, ϵ', μ' are the real parts and ϵ'', μ'' are the imaginary parts of the relative permittivity and permeability, respectively. The real part takes into account the capacity to store the electric/magnetic energy whereas imaginary part takes into account dielectric/magnetic energy losses. It is well known fact that dielectric-magnetic medium is simply described through its permittivity and permeability whereas a reciprocal chiral medium has three constitutive parameters; permittivity, permeability and chirality [8,9]. It is described that, instead of incorporating all properties of certain medium through the constitutive parameters one can additionally introduce the non-integer dimensional (NID) space to incorporate some properties.

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NID space is now a popular topic in physics and engineering [10,11]. Axiomatic basis for space had been formulated by Stillinger [12]. In recent years, Tarasov had given several significant contributions incorporating the concept of NID space [13–15]. Equation of motion in a NID space had been develop in [16]. Concept of NID space may be directly managed through the generalized definition of operators ∇_D, $\nabla_D\times$, ∇_D^2, where subscript D has been used to describe the dimension of space [17]

$$\nabla_D = \left(\frac{\partial}{\partial x}+\frac{\alpha_1-1}{2x}\right)\hat{x}+\left(\frac{\partial}{\partial y}+\frac{\alpha_2-1}{2y}\right)\hat{y}+\left(\frac{\partial}{\partial z}+\frac{\alpha_3-1}{2z}\right)\hat{z}$$

$$\nabla_D^2 = \frac{\partial^2}{\partial x^2}+\frac{\alpha_1-1}{x}\frac{\partial}{\partial x}+\frac{\partial^2}{\partial y^2}+\frac{\alpha_2-1}{y}\frac{\partial}{\partial y}+\frac{\partial^2}{\partial z^2}+\frac{\alpha_3-1}{z}\frac{\partial}{\partial z}$$

$$\begin{aligned}\nabla_D\times = &\left\{\left(\frac{\partial}{\partial y}+\frac{\alpha_2-1}{2y}\right)-\left(\frac{\partial}{\partial z}+\frac{\alpha_3-1}{2z}\right)\right\}\hat{x}\\ &+\left\{\left(\frac{\partial}{\partial z}+\frac{\alpha_3-1}{2z}\right)-\left(\frac{\partial}{\partial x}+\frac{\alpha_1-1}{2x}\right)\right\}\hat{y}\\ &+\left\{\left(\frac{\partial}{\partial x}+\frac{\alpha_1-1}{2x}\right)-\left(\frac{\partial}{\partial y}+\frac{\alpha_2-1}{2y}\right)\right\}\hat{z}\end{aligned}$$

where parameters $0<\alpha_1<1$, $0<\alpha_2<1$, $0<\alpha_3<1$ are measurement distributions on coordinates x, y and z, respectively. The total dimension of the space is $D=\alpha_1+\alpha_2+\alpha_3$. The solutions of the following homogeneous Helmholtz's equation

$$\nabla_D^2 E(x,y)+k^2E(x,y)=0$$

in the medium having NID only along y-axis are [17,18]

$$E(x,y)=\exp\left(\pm i\frac{\xi x}{a}\right)\left(\frac{\eta y}{a}\right)^n H_n^{(2)}\left(\frac{\eta y}{a}\right)$$

$$E(x,y)=\exp\left(\pm i\frac{\xi x}{a}\right)\left(\frac{\eta y}{a}\right)^n H_n^{(1)}\left(\frac{\eta y}{a}\right)$$

where $\xi=k_x a$, $\eta=k_y a$, $k=\sqrt{k_x^2+k_y^2}$, $n=\frac{3-D}{2}$, $1\le D\le 2$. Recently Abbas et al. has contributed on radiation of cylindrical wave in NID space [19]. Scattering from a low contrast circular cylinder has been contributed by Abbas et al.[20]. In 1931, Kobayasi introduced a new analytical method to treat potential problems associated with single or two disks [21]. Later, Sneddon used the name Kobayasi Potential (KP), for this method, in his book [22]. KP method is a semi-analytic method [23–25] used to investigate scattering and diffraction problems. Using this method various kinds of problems such as diffraction of electromagnetic plane wave by rectangular plate and hole [26], parallel slits [27], disk and circular hole [28], perfect electric conducting (PEC) strip [29,30] had been addressed. Scattering of a plane wave by a buried conducting strip had been investigated by Hongo and Hamamura [31]. In present manuscript, Kobayasi potential method has been used to investigate the scattering of electromagnetic plane wave by a cylindrical obstacle deeply buried beneath a NID dielectric slab. For zone scattered field has been derived assuming no interaction of the scattered field with the strip when it reflects from the slab.

2. Formulation

Consider the geometry for scattering problem as shown in Fig. 1. It contains a perfect electric conductor (PEC) strip which is placed under an infinite extent planar dielectric slab. The wave number of medium filling the slab is $k=\omega\sqrt{\mu\epsilon}$ whereas medium hosting the slab has wave number $k_0=\omega\sqrt{\mu_0\epsilon_0}$. It is assumed that slab is NID along y direction of the coordinate system. One interface of the planar slab is located at $y=b_1$ whereas other interface is located at $y=b_2>b_1$. Thickness of the strip is negligibly small and its width is $2a$. The length of the strip is infinite along z-direction of the cartesian coordinate axes. It has also been assumed that strip is deeply buried under the NID slab. The geometry has been excited by a transverse electric uniform plane wave which makes an angle α with x-axis. The time dependency is taken to be $\exp(-iwt)$ throughout this work.

The problem is decomposed into two parts. In the first part, it is assumed that strip is absent. So fields inside and outside the slab, due to plane wave excitation, are written in terms of unknowns. These unknowns are determined using the boundary conditions at interfaces of the slab. In second part of the problem, presence of the strip is taken into account by taking transmitted field as excitation on the strip. It has been assumed that spacing between the strip and slab is very large as compared to the wavelength, so scattered field after reflection from the slab has no interaction with the strip. Due to this assumption, term deeply buried cylindrical obstacle has been used.