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Real-time wavefront reconstruction for extended object based on phase diversity

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ABSTRACT

We present a real-time solution to the phase diversity problem when the observed objects are extended scenes. It utilizes an iterative linearization of the optical transfer functions (OTF) in at least two diversity planes by a first-order Taylor expansion to reconstruct initial wavefront. Vast simulation experiments are processed to verify the presented algorithm, including comparing our algorithm with the analytic estimator method, demonstrating that our method has high wavefront detection accuracy and large linearity range.

Introduction

Since the inception of phase diversity (PD) about two decades ago, many authors have used the PD method [1–5], which utilizes at least two images of the same object recorded in presence of a known optical aberration (e.g. defocus), to estimate the wavefront of optical systems and enhance the detected image. The hardware of PD technique is limited to or can be merged in the usual imaging sensor, the number of estimated modes can be continuously tuned and it is among the very few methods enabling the measurement of differential pistons, tip-tilts on segmented or divided apertures. However, the main drawback of classic PD is that since it is a nonlinear optimization problem, complexity is reported on data processing. Due to the high computational complexity and possible convergence to local optima [6], the nonlinear PD has a limited usage in real-time correction algorithms.

Considerable effort has been directed toward decreasing the computational complexity of the PD algorithm. The common idea is to linearize the generalized pupil function (GPF) or point spread function (PSF) based on the assumption that the total aberration is small, such as paper [7–9] utilize an expansion of the GPF to retrieve the unknown phase and paper [10,11] linearize PSF to reconstruct the wavefront. But all of them need a single point source, and thus cannot be operated on observation systems which take images of very extended scenes. Paper [12] presented an approximation of optical transfer function (OTF) which can be used for the detection of wavefront aberrations on extended object, but is limited to extremely

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small phase aberrations. In this letter, we present a real-time wavefront reconstruction for extended object by the use of the iterative linearization of OTFs in at least two diversity planes and compare with the analytic estimator method proposed in paper [12]. The experimental results demonstrate that our method effectively increases the linearity range, making it much larger than that in paper [12].

We first present the model for image formation through the optical system in the presence of aberrations. The phase aberrations $\phi_k \in \mathbb{R}^{m^2\times 1}$ in the k-th diversity image can be approximated using a normalized Zernike basis [13]:

$$\phi_k(u_j, v_j) = Z(u_j, v_j)(\alpha + \beta_k), \tag{1}$$

where m^2 is the number of pixels, $\alpha \in \mathbb{R}^{n\times 1}$ is the n Zernike coefficients to be reconstructed, $\beta_k \in \mathbb{R}^{n\times 1}$ is the introduced known diversity to eliminate the ambiguity of the inverse problem, $Z(u_j, v_j) \in \mathbb{R}^{m^2\times n}$ is the matrix of the n Zernike polynomials evaluated in the pupil plane (u_j, v_j) coordinates. These phase aberrations nonlinearly influence the PSF expressed in the spatial coordinates (s_j, t_j), which can be written as:

$$h(s_j, t_j; \alpha, \beta_k) = \left|\Im\left[P(u_j, v_j)\exp(i\phi_k(u_j, v_j))\right]\right|^2(s_j, t_j), \tag{2}$$

where $\Im$ is the Fourier transform and P is the binary pupil function.

The image recorded at the k-th optical plane of an instrument is modeled by the discrete and noisy convolution of the PSF with the observed object, shown as:

$$y_{k,j}(s_j, t_j) = o * h(s_j, t_j; \alpha, \beta_k) + n_k(s_j, t_j), \tag{3}$$

where $y_{k,j}$ denotes the j-th pixel of the k-th diversity image, o is the true observed object and $n_k(s_j, t_j)$is additive noise. Eq. (3) can be rewritten in frequency domain as below:

$$Y_{k,j}(f_{s_j}, f_{t_j}) = \mathrm{O} \bullet \mathrm{S}(f_{s_j}, f_{t_j}; \alpha, \beta_k) + N_k(f_{s_j}, f_{t_j}), \tag{4}$$

where $Y_{k,j}$, O, S and N_k are discrete Fourier transforms of $y_{k,j}$, o, h and n_k, respectively, (f_sj, f_tj) is the coordinate vector in the frequency domain and $\bullet$ denotes the dot product. We introduce the short-hand notations of $S(f_sj, f_tj; \alpha, \beta_k)$as $S_j(\alpha, \beta_k)$, recall that $S_j(\alpha, \beta_k)$ is the OTF of the k-th image plane.

We approximate the OTF by a first order Taylor expansion for small aberrations and non-zero diversities. The first-order Taylor approximation of the OTF in $\alpha = 0$ is given by:

$$S_j(\alpha, \beta_k) = D_{0,j}(\beta_k) + D_{1,j}(\beta_k)\alpha + O\|\alpha\|^2, \tag{5}$$

where$D_{0,j}(\beta_k) = S_j(\alpha, \beta_k)|_{\alpha=0}$, $D_{1,j}(\beta_k) = \frac{\partial S_j(\alpha,\beta_k)}{\partial\alpha}|_{\alpha=0}$ and $O\|\alpha\|^2$is the 2-th order Lagrange residue.

In order to eliminate the unknown object to establish a direct relationship between the detected images and the unknown aberrations, two images Y_1 and Y_2 are used here based on PD technique. Multiplying them by each other's OTF, thus we get Eqs. (6) and (7):

$$S_2 \bullet Y_1 = S_2 \bullet O \bullet S_1 + S_2 \bullet N_1, \tag{6}$$

$$S_1 \bullet Y_2 = S_1 \bullet O \bullet S_2 + S_1 \bullet N_2. \tag{7}$$

Since these two equations only involving dot product, we can eliminate the unknown object by subtracting these two image measurements, shown as below:

$$S_2 \bullet Y_1 - S_1 \bullet Y_2 = S_2 \bullet N_1 - S_1 \bullet N_2. \tag{8}$$

Substituting Eq. (5) into this expression and abandoning 2-th order residue yields a new estimate of phase via the solution of a least-square (LS) problem:

$$Y = A\alpha + \Delta N, \tag{9}$$

where

$Y = D_{0,j}(\beta_1) \bullet Y_2 - D_{0,j}(\beta_2) \bullet Y_1$, $A = D_{1,j}(\beta_2) \bullet Y_1 - D_{1,j}(\beta_1) \bullet Y_2$, $\Delta N = S_1 \bullet N_2 - S_2 \bullet N_1$

Thus, the linear estimate of the phase aberrations can be obtained by the LS estimator:

$$\alpha = \left[\Re(A^T A)\right]^{\dagger}\left[\Re(A^T Y)\right], \tag{10}$$

where $\bullet^T$ denotes the transposition, $\Re$ represents the real part operator and $\bullet^{\dagger}$ is the generalized inverse of a matrix.

We consider only the static setting, where the aberrations do not change in the time window considered. We start with an initial estimate of α using the OTF approximation around zero aberration, then a new linearization of the OTF can be done around the current phase estimate $\hat{\alpha}$ and a new least squares (LS) problem can be established to solve the next estimate. We repeat the process until reach the number of iteration times or the norm of the aberration increment reach the set threshold,