Optik 144 (2017) 436–445

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

# A theoretical analysis for mean exit time of a Bi-stable system under combined Gaussian and Poisson white noise excitations

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ARTICLE INFO

*Article history:*
Received 10 April 2017
Accepted 3 July 2017

*Keywords:*
Perturbation scheme
Gaussian white noise
Poisson white noise
Laplace integral method
Mean exit time

ABSTRACT

In this work, an approximate method is proposed to simultaneously compute an analytic solution for the mean exit time statistics in a bi-stable system with combined Gaussian and Poisson white noise excitations. Firstly, the method employs a perturbation scheme coupled with the Laplace integral method to estimate the mean exit time analytically. Then, a bi-stable system under both external and parametric excitations of combined Gaussian and Poisson white noises are investigated illustratively with the proposed technique, and effects of noise intensity and random magnitude of impulses on the mean exit time are discussed respectively. Finally, the effectiveness of the theoretical results is verified by means of Monte Carlo simulation, and good agreement will be observed.



## 1. Introduction

Gaussian white noise provides efficient and often useful models of various environmental loading, and a broader class of random processes, like, Poisson processes, presents a more realistic description to the disturbances of discrete events. It has been widely adopted and studied in many aspects. Such as, range of loadings, including earthquake, wave and traffic, as well as wind. However, in a variety of engineering and science problems, the linear and nonlinear systems are often imposed with combinations of continuous random processes and random pulses, as a peculiar case. In recent years, a number of investigators have concentrated on the research of systems under combined excitations of Gaussian and Poisson white noises. Kumar and Popovic [1] obtained large deviation results for a two time-scale model dynamical systems under combined excitations of Gaussian and Poisson white noises. Pirrotta and Santoro [2] studied probabilistic responses of nonlinear systems under combined Gaussian and Poisson white noises via the path integral method. Liu and Zhu [3] applied the Lyapunov function method to find stability analysis of quasi-Hamiltonian systems under combined Gaussian and Poisson white noise excitations. Zhu et al. [4] obtained stationary probability density function solutions of stochastic responses of nonlinear oscillators subjected to combined Gaussian and Poisson white noises by the exponential-polynomial closure method. Potrykus et al. [5] developed practical computational methods to obtain response statistics of linear systems under combined excitations of Gaussian and Poisson white noises. Sun and Duan et al. [6] considered a stochastic differential equation model for nonlinear oscillators under combined excitations of Gaussian and Poisson white noises.

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http://dx.doi.org/10.1016/j.ijleo.2017.07.007
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The exit problem has attracted considerable interests over the past decades [7–10]. The exit phenomenon can be defined as, the process from a bounded domain in state space, which is the result of the evolution of the dynamical system. Mean exit time (MET) is a quantitative measure of the exit phenomenon, and it is the mean time of a particle in a bi-stable system exiting from one state to another one [11,12]. Many works on the MET in dynamic systems have been carried out, but only the case of Gaussian or Poisson white noise excitation was considered. Freidlin and Wentzell [13] used large-deviation theory to obtain exit probability of stochastic dynamic system with Gaussian white noise. Han et al. [14] obtained MET statistics in a bi-stable system with Poisson white noise by using the generalized cell mapping method. Grigoriu [15] obtained reliability and exit probability of linear dynamic systems with Poisson white noise by conditional Monte Carlo simulation. Zeng et al. [16] considered responses and first-exit failure of hysteretic systems under non-Gaussian random excitation by Monte Carlo simulation. Then in this present paper, we mainly are to consider the MET of a system under combined Gaussian and Poisson white noises excitations.

The MET of the system under Poisson white noise are mainly based on numerical analysis. For example, Atiya et al. [17] developed a fast Monte Carlo-type numerical methods for computer the density function of MET for dynamic systems under combined Gaussian and Poisson white noises excitations. Kim et al. [18] proposed the iterative algorithm for the distribution of MET in a dynamic system under combined Gaussian and Poisson white noises excitations. But, an approximate theoretical solution to MET will help us to gain a better insight into the exit time. However, due to the excitation of Poisson white noise, the equation of MET for nonlinear systems to Gaussian and Poisson white noises will be an infinite order partial differential equation, that is rather hard to theoretically solve. Therefore, this paper aims to propose an approximate theoretical solution to MET for a one-dimensional bi-stable system [19] under combined excitations of Gaussian and Poisson white noises by means of perturbation scheme and Laplace integral method [20].

The organization of this paper is as follows. In Section 2, we introduce the definition of MET for a one-dimensional bi-stable system under combined Gaussian and Poisson white noise excitations. In Section 3, the equation to describe MET of a bi-stable system is presented. In Section 4, an approximate theoretical solution to MET of the bi-stable system under combined Gaussian and Poisson white noises is obtained by perturbation technique. In Section 5, two examples, as bi-stable systems under both external and parametric excitations of combined Gaussian and Poisson white noises are worked out in detail to illustrate the application of the proposed method, meanwhile, all of the analytic results are confirmed by Monte Carlo simulations. The paper is concluded in Section 6.

## 2. The definition of MET

Consider the following stochastic differential equation

$$\frac{dx}{dt} = f(x,t) + g(x,t) W_B(t) + h(x,t) W_P(t), \tag{1}$$

where the $f(x,t) = -V'(x,t)$, $V(x,t)$ is a double well potential and the prime denotes the spatial derivative. $W_P(t)$ is Poisson white noise, and $W_B(t)$ is Gaussian white noise with zero mean and correlation functions $\mathrm{E}[W_B(t) W_B(t+\tau)] = 2D\delta(\tau)$,$\mathrm{E}\{\cdot\}$ denotes the expectation, and $W_B(t)$, $W_P(t)$ are assumed to be mutually independent. The Gaussian and Poisson white noise are defined by the formal derivatives of the Wiener process and compound Poisson process, which are special forms of Lévy process [21–23]

$$W_B(t) = \frac{dB(t)}{dt}, W_P(t) = \frac{dC(t)}{dt},$$

$$C(t) = \sum_{k=1}^{N(t)} Y_k U(t - t_k),$$

where $\delta(\cdot)$ is the Dirac delta function, $U(t)$ is the unit step function.$N(t)$ is a Poisson counting process with the mean arrival rate $\lambda$, representing the number of impulses in the time interval $[0, t)$. $Y_k$ are independent identically distributed random variables, which are independent of the impulse arrival time $t_k$, when $\lambda \to \infty$, at the same time, the noise intensity $I_0 = \lambda \mathrm{E}\left[Y^2\right]$ keeps a constant value, the Poisson white noise $W_P(t)$ approaches to a Gaussian one [24]. The Wiener and Poisson processes form the tools of a toolbox to create jump-diffusion process models [25]. The Poisson process $W_P(t)$ supplies the jumps and the Wiener process $W_B(t)$ supplied the diffusion.

For a bounded domain $\Gamma$, we define the first exit time from $\Gamma$ with starting from $y \in \Gamma$ at time $t_0$ as follows [26]

$$\tau_\Gamma(\omega) := \inf\left\{t > 0 : x_0 = y, x_t \in \partial\Gamma\right\}.$$

It is known that $\tau_\Gamma$ is a stopping time. The MET is denoted as

$$u(y) := \mathrm{E}\left[\tau_\Gamma(\omega)\right].$$

It is the mean exit time of a particle initially at $y$ inside $\Gamma$until the particle first hits the boundary $\partial\Gamma$.