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Original research article

Propagation of transverse magnetic mode in a non-integer dimensional dielectric slab waveguide

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ABSTRACT

In present paper, solutions for the transverse magnetic modes propagating in a non-integer dimensional (NID) dielectric slab waveguide have been derived. Medium within the slab is NID in direction normal to the dielectric interfaces of the slab. Transcendental equations in terms of non-integer order Hankel functions have been derived. The order of non-integer order Hankel functions depends on parameter describing dimension of the NID space. Using graphical treatment, it has been shown that behavior of propagating mode changes substantially when value of the parameter describing dimension of the NID space changes. It has also been noted that NID parameter significantly effects the value of propagation constant, attenuation constant and cutoff frequency. The classical results have also been recovered when dimension of the NID slab has been taken equal to the dimension of the medium surrounding the slab.

1. Introduction

Objects with irregular shape like dust particles, clouds, mountains and ocean waves etc., cannot be explained by Euclidean geometry. To address these type of geometries/objects, the idea of fractals was introduced [1,2]. Mandelbrot [3] was the first who developed proper classification of mediums into fractals and non-fractals. Solving research problems using approach of non-integer dimension is a popular activity in research [4–7]. Non-integer dimensional (NID) space may be used to address fractal medium and it has gained attention of research community belonging to disciplines of physics and engineering [8–10]. The idea of mathematical basis for NID space was proposed by Stillinger [11]. Sandev et al. [12] studied quantum mechanical oscillator in NID space. The concept of equations of motion in NID space was given by Palmer and Stavrinou [13]. The propagation of plane, cylindrical and spherical electromagnetic waves in NID space was given by Zubair et al. [14–16]. Recently, Mughal and Zubair worked on antenna radiation in NID space [17]. Electromagnetic fields in NID space was also analyzed by Tarasov [18]. He had contributed lot by studying problems, related to various topic, in NID space [5,19–25]. This approach had also been used to derive the Green's function for NID dielectric half space geometry [26].

Martin et al. [27] reformulated Faraday's law, Ampere's law and Maxwell equation using tensor algebra for NID space. Also solution of Laplace and Poisson's equation in non-integer dimensional space had been derived [28,29]. The transmission and reflection coefficients for a dielectric slab filled with NID dielectric slab had been derived by Asad et al. [30]. Many researchers have worked to explain other physical phenomena in NID space [31–34].

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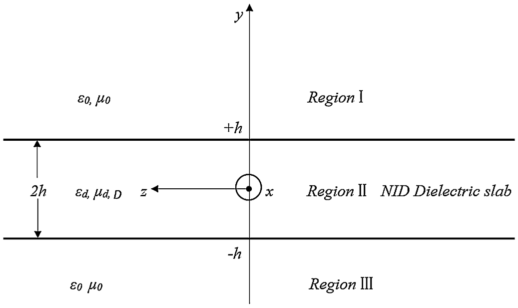

Fig. 1. Non-integer dimensional dielectric slab waveguide.

In present work, general solution for TM^z mode propagating in a NID dielectric slab waveguide has been treated. In first section, geometry of the problem has been described. Section 2 deals with analytical derivation of the general solution for TM^z mode. In this section, expressions of electric and magnetic fields for NID dielectric waveguide have been derived using the axial component of magnetic vector potential. Treatment has been divided into odd and even modes and solutions for both cases have been derived. Section 3 deals with a the graphically method to get solution. Behavior of odd and even modes in NID slab waveguide has been noted for different values of the parameter describing dimension of the NID space. Results given in published literature have also reproduced as special case.

2. Geometry of the problem

Consider a planner NID dielectric waveguide geometry shown in Fig. 1. It contains a planner dielectric slab having infinite extent and is placed in free space. Thickness of the slab is $2h$. Permitivity and permeability of the medium filling the dielectric slab are represented by ϵ_d and $\mu_d = \mu_0$, respectively. It has been assumed that dielectric medium filling the slab is NID and the dimension of the medium is represented by D. It has also been assumed that the space within slab is NID only along y-direction. For the sake of simplicity it has been assumed that field/potential quantities are independent of x, that is, $\frac{\partial}{\partial x} = 0$. The whole geometry is divided into three regions, region I ($y \geq h$) is the outside the dielectric slab, region II ($-h \leq y \leq h$) represents the inner side of the dielectric slab and region III ($y \leq -h$) is the below the dielectric slab.

It is a well known fact that a waveguide is designed to guide the contained energy within the structure. When the incidence angle is greater than critical angle the wave bounce back and forth between upper and lower interfaces of the guide. Refracted field outside the slab may also exist in form of decaying waves and all the real energy remains within the slab. Characteristics of the guiding structure can be analyzed by assuming modal solutions of the Helmholtz's equation in different regions of the geometry. Unknown quantities in the assumed solution can be determined by using the boundary conditions related to the geometry. In the next section, propagation of TM^z modes inside a NID dielectric slab waveguide are derived.

3. Transverse magnetic (TM^z) modes

When electromagnetic modes in a waveguide satisfy condition $H_z = 0$, it is termed as TM^z mode. Solution for the TM^z mode may be derived by considering solution of the homogeneous Helmholtz's equation for z-component of the magnetic vector potential **A**.

In region I and III of the above geometry, z-component of magnetic vector potential A_z, must satisfy the following Helmholtz's equation:

$$\nabla^2 A_z(x, y, z) + \beta_0^2 A_z(x, y, z) = 0 \tag{1}$$

where $\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$. It may be noted that $\mathbf{A(x, y, \hat{z}) = zA_z(y, z)}$. Using separation of variables method, solution of (1) for region I may be assumed as written below

$$A_z^{0+} = B_{me}^{0+} \exp\left\{-j(\beta_{0y}y + \beta_z z)\right\} + A_{mo}^{0+} \exp\left\{-j(\beta_{0y}y + \beta_z z)\right\} = A_{ze}^{0+} + A_{zo}^{0+} \tag{2}$$

where $\beta_0^2 = \omega^2\mu_0\epsilon_0 = \beta_{0y}^2 + \beta_z^2$. In above equation, B_{me}^{0+} and A_{mo}^{0+} are unknowns to be determined. Superscript 0+ stands for free space region above the slab whereas subscript e and o stand for even and odd modes, respectively. For evanescent mode $\beta_{0y} =- j\alpha_{0y}$ and α_{0y} is real quantity. Therefore

$$A_z^{0+} = B_{me}^{0+} \exp\left(-(\alpha_{0y}y + j\beta_z z)\right\} + A_{mo}^{0+} \exp\left\{-(\alpha_{0y}y + j\beta_z z)\right\} = A_{ze}^{0+} + A_{zo}^{0+} \tag{3}$$

The condition for evanescent mode yields

$$\beta_{0y}^2 + \beta_z^2 = -\alpha_{0y}^2 + \beta_z^2 = \beta_0^2 \tag{4}$$