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On the unsteady and time-harmonic conservation laws of electromagnetic energy and chirality

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ABSTRACT

We examine conservation laws of the energy and chirality for electromagnetic waves in the case of loss-free dielectric media. We show that the energy allows for conservation laws for both generic unsteady and time-harmonic fields. In comparison, the chirality admits a conservation law only for time- harmonic fields. This difference in the time dependence illustrates the crucial distinction between the energy consisting of scalar products of field variables and the chirality composed of their vector products. For future extension of our analysis, we derive those conservation laws for spatially inhomogeneous refractive index. As a result, we uncover physical implications of the new terms in the conservation laws, that have not been considered in the conventional literature.

1. Introduction

Electromagnetic (EM) waves are characterized by a variety of properties, that are bilinear in the field variables. As one of such properties, the energy density is the most important charactreistics of EM waves. Notwithstanding, the EM energy density as squared manitudes of the field vectors cannot properly reveal the phase relations between the electric and magnetic fields. Complementary to the EM energy, the EM chirality accounts for the phase difference between the electric and magnetic fields [1–10].

While the orbital and spin parts of angular momentum constituting the energy-current (Poynting) vector are relevant to numerous application areas including manipulation of nano-objects [3,11,12], the EM chirality is related to the application areas including, for instance, the enantiomer separation [2,9]. Notice that chirality is sometimes referred to as helicity [8]. It is well known that both energy and chirality satisfy respective conservation laws. In fact, energy and chirality are associated with each other through such conservation laws.

When it comes to the temporal features of EM waves, it is however not quite certain whether both energy and chirality satisfy such conservation laws for generic unsteady (viz., non-periodic) fields, which include, for instance, optical pulses [3]. Only for time-harmonic fields, several common features hold true to the conservation laws of energy and chirality. For this matter, we will examine this issue in this study by trying to establish conservation laws for both unsteady and time-harmonic fields.

Both energy and chirality of EM waves have been investigated largely for vacuum or homogeneous dielectric media. Therefore, we try in this study to establish conservation laws for inhomogeneous media [13–21]. Recent advances in gradi-

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ent metamaterials and in a broader sense in bulk gradient metamaterials spur us to investigate inhomogeneous media [20]. For instance, we find that plasmonic-enhanced inhomogenoeus laser fields are useful for studying atomic physics, where the spatial inhomogeneity of the field is sufficiently fine enough on the nanometer scale to get into interactions with the atomic processes [21]. With modern nano-fabrication technologies at hand, metamaterials with a variety of artificial spatial inhomogeneities are increasingly available [18]. Our study serevs as a starting point for dealing with such spatial inhomogeneities from the standpoint of two important conservation laws of EM waves. However, how to practically achieve the spatial variations in the refractive indices are out of scope of this study.

As an additional example, where unsteady fields do not work properly but time-harmonic fields work fine, we derive conservation laws of both energy and chirality for gyrotropic media [22–25]. Based on our recent studies on EM waves [26–28], we hope to expand further on their angular momentm and spin in the near future particularly for inhomogeneus media. To this goal, we will provide full details of all the pertinent derivations to make our study self-contained and serve as a reference for future endeavors.

2. Basic formulation

In SI units, the Maxwell's equations are given as follows [9].

$$\begin{cases} \tilde{\nabla} \times \vec{\mathscr{B}} = n^2 \tilde{\varepsilon}_0 \dfrac{\partial \vec{\mathcal{A}}}{\partial \tilde{t}} \\ \tilde{\nabla} \times \vec{\mathcal{A}} = -\tilde{\mu}_0 \dfrac{\partial \vec{\mathscr{B}}}{\partial \tilde{t}} \end{cases}, \quad \begin{cases} \tilde{\nabla} \cdot \left(n^2 \tilde{\varepsilon}_0 \vec{\mathcal{A}}\right) = 0 \\ \tilde{\nabla} \cdot \vec{\mathscr{B}} = 0 \end{cases}. \tag{2.1}$$

Here, the first and second equations are the Ampère's circuital law and the Faraday's law of induction in the absence of electric space charge. In addition, the third and fourth equations are the electric Gauss's law for electric displacement and the magnetic Gauss's law in the absence of magnetic monopoles [4].

Concerning notations, $\vec{\mathcal{A}}$ and $\vec{\mathcal{B}}$ are dimensional and the electric and magnetic fields, respectively. Both are real and space vectors, namely, $\vec{\mathcal{A}}, \vec{\mathcal{B}} \in \mathbb{R}^3$ with $\mathbb{R}$ denoting real space. Furthermore, $\tilde{t}$ is the dimensional time, whereas $\tilde{\nabla}\times$ and $\tilde{\nabla}\cdot$ are the usual dimensional differential operators for curl and divergence, respectively. Both space derivatives are based on the dimensional coordinates $(\tilde{x}, \tilde{y}, \tilde{z})$ in the Cartesian coordinate system.

As the principal material properties of EM waves, the dimensional parameters $\tilde{\varepsilon}_0$ and $\tilde{\mu}_0$ with $\tilde{\varepsilon}_0, \tilde{\mu}_0 \in \mathbb{R}$ are electric permittivity and magnetic permeability, respectively. Non-magnetic media are assumed throughout this study. We assume the media to be dielectric and loss-free [22]. Therefore, the refractive index n is real and dimensionless so that $n \in \mathbb{R}$. It is greater than unity, namely, $n \geq 1$. Its square n^2 is hence the relative electric permittivity ε such that $\varepsilon \equiv n^2$ [4,5,19]. In general, we assume n to be spatially inhomogeneous, namely, $n = n(\tilde{x}, \tilde{y}, \tilde{z})$.

Let us introduce another pair of real dimensional field vectors $\vec{\mathcal{X}}$ and $\vec{\mathcal{H}}$ with $\vec{\mathcal{X}}, \vec{\mathcal{H}} \in \mathbb{R}^3$ such that they normalize $\vec{\mathcal{A}}$ and $\vec{\mathcal{B}}$ as follows.

$$\sqrt{\tilde{\varepsilon}_0}\vec{\mathcal{A}} \equiv \vec{\mathcal{X}}, \quad \sqrt{\tilde{\mu}_0}\vec{\mathcal{B}} \equiv n_{ref}\vec{\mathcal{H}}. \tag{2.2}$$

Here, n_{ref} is a prescribed refractive index so that it is dimensionless and $n_{ref} > 0$. In this aspect, notice that n_{ref} is not necessarily identical to $n_0 = 1$ for vacuum. For simplicity of notation, we define the squared and normalized refractive index $\mathcal{N} \equiv \left(n/n_{ref}\right)^2$, whereby $0 \leq \mathcal{N} < \infty$ in principle.

Let us now introduce additional reference parameters by considering a monochromatic wave with a certain fixed frequency $\tilde{\omega}_0$ with $\tilde{\omega}_0 > 0$ [9]. A natural choice of reference time is $\tilde{t}_{ref} \equiv \tilde{\omega}_0^{-1}$ so that a dimensionless time is defined by $t \equiv \tilde{\omega}_0 \tilde{t}$. Notice here that we are still dealing with generic unsteady field variables. Turning now to the space coordinates, we notice that the speed of light is defined by $\tilde{c}_0$ in vacuum. Further define the reference length by $\tilde{l}_{ref}$ and the reference wave number by $\tilde{k}_{ref} \equiv \tilde{l}_{ref}^{-1}$. In summary for the reference parameters,

$$\tilde{c}_0 \equiv \frac{1}{\sqrt{\tilde{\varepsilon}_0 \tilde{\mu}_0}}, \quad \tilde{t}_{ref} \equiv \frac{1}{\tilde{\omega}_0}, \quad t \equiv \tilde{\omega}_0 \tilde{t}, \quad \tilde{l}_{ref} \equiv \frac{\tilde{c}_0}{n_{ref}\tilde{\omega}_0}. \tag{2.3}$$

With all these dimensionless variables and parameters, the Maxwell's equations in Eq. (2.1) are reduced to the following normalized dimensionless set.

$$\begin{cases} \nabla \times \vec{\mathcal{H}} = \mathcal{N} \dfrac{\partial \vec{\mathcal{X}}}{\partial t} \\ \nabla \times \vec{\mathcal{X}} = -\dfrac{\partial \vec{\mathcal{H}}}{\partial t} \end{cases}, \quad \begin{cases} \nabla \cdot \left(\mathcal{N}\vec{\mathcal{X}}\right) = 0 \\ \nabla \cdot \vec{\mathcal{H}} = 0 \end{cases}. \tag{2.4}$$

Notice that the dimensionless coordinates are now defined by $(x, y, z) \equiv \tilde{k}_{ref}(\tilde{x}, \tilde{y}, \tilde{z})$ via Eq. (2.3). Accordingly, the dimensionless operators $\nabla\times$ and $\nabla\cdot$ are constructed via these dimensionless coordinates. In this study, we are however interested in discovering what happens to non-vacuum dielectric inhomogeneous media with non-constant $\mathcal{N}(x, y, z)$. Let us stress that both $\vec{\mathcal{X}}$ and $\vec{\mathcal{H}}$ are physical field vectors.