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Energy flux density and angular momentum density of radial Pearcey-Gauss vortex array beams in the far field

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ABSTRACT

The longitudinal and transverse energy flux density (EFD) and angular momentum density (AMD) of the radial Pearcey-Gauss vortex array beams are studied by using the vector angular spectrum representation and stationary phase method, where the influence of topological charge, initial phase index, noncanonical strength and the number of beamlet on far-field vectorial structures of the corresponding beam is emphasized. The results show that the topological charge of center optical vortex is always equal to initial phase index for different noncanonical strength. The longitudinal EFDs exhibit different structures by varying initial phase index and the number of beamlet. For an odd-*N*, the topological charge of center optical vortex can be determined by distinguishing the longitudinal EFD, however for an even-*N* the topological charge cannot be identified due to the equivalence of longitudinal EFD in the case of positive and negative initial phase index *m*. The AMD tends to split more branches from the positive and negative direction with increasing the absolute value of topological charge *l*, The inner structures of AMD are also affected by different beamlets while the zero value of AMD is always retained.

1. Introduction

In recent years there has been considerable interest in exploring the wave propagation and topological dynamics of laser beams with catastrophe functions, where the catastrophe functions are generally associated with the oscillatory cuspoid canonical integrals and the integrals are defined by [1,2]

$$C_m(a) = \int_{-\infty}^{\infty} \exp\left[iP_m(a,u)\right] du, \ \ P_m(a,u) = u^m + \sum_{j=1}^{m-2} a_j u^j. \tag{1}$$

The oscillatory integral $C_m(a)$ plays an important role in the propagation properties and topological dynamics of laser beams. The simplest case is the fold catastrophe for $m=3$, which is proportional to the well-known Airy function. The truncated Airy beams have exhibited the intriguing properties of self-healing, self-acceleration, plasma guidance, sorting micro-particles and electron acceleration since the seminal work of Siviloglou et al. in 2007 [3–11]. The next more complicated case is cusp catastrophe for $m=4$, namely Pearcey function [12]. The Pearcey beam presents its auto-focusing,

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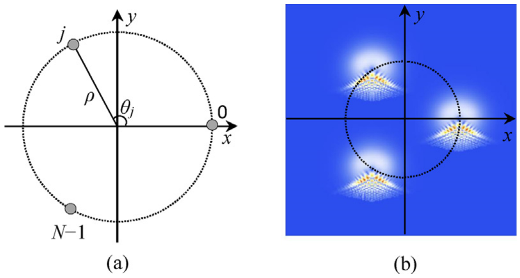

Fig. 1. Sketch of radial PeG vortex array beams formed by $N=3$ identical off-axis PeG vortex beamlets at $z=0$.

self-healing and form-invariance properties during propagation by introducing the Pearcey function to optics in 2012 [13]. Furthermore, these investigations are also extended to Half-Pearcey beams and Pearcey-Gaussian vortex beams [14,15].

On the other hand, based on the vector angular spectrum methods the general solution of Maxwell's equation is composed of transverse electric mode (TE) and transverse magnetic mode (TM) terms of electromagnetic field [16]. Much effort has been devoted to investigating the far-field vectorial structures of various beams such as Gaussian, hollow Gaussian, Lorentz-Gauss, Laguerre-Gauss, Airy, vortex Airy and combined Airy vortex array beams [17–23]. As the simplest case of catastrophe optics, an Airy beam represents different far-field divergent properties in the transverse plane [20]. Chen et al. studied the effect of optical vortex on the far-field vectorial structures of Airy beam [21]. But we cannot find more details about the far-field angular momentum density (AMD) of radial array beams. It is natural to ask whether the radial Pearcey-Gauss (PeG) vortex array beam shows distinctive properties especially in term of the EFD and AMD. The purpose of this paper is to explore the longitudinal and transverse EFDs and AMDs of radial PeG vortex array beams formed by the radial coherent combination of N identical off-axis PeG vortex beamlets. The results obtained in radial PeG vortex array beams are useful for determining the orbital angular momentum of radial array beams.

2. Far-field vectorial structures of radial pearcey-Gauss vortex array beams

Assume that N identical off-axis PeG vortex beamlets are uniformly distributed along the radial direction as depicted in Fig. 1, where each beamlet linear-polarized in the x-direction is symmetrically located on a circle with radius ρ and carries a different initial phase φ_j. The off-axis coordinates of the j-th beamlet are $x_j=\rho\cos\theta_j$ and $y_j=\rho\sin\theta_j$ along x and y axes at $z=0$, respectively, where the azimuth angle of the j-th beamlet is $\theta_j=2j\pi/N$ [24].

In the Cartesian coordinate system the linear-polarized x-direction electric field of the input j-th off-axis PeG vortex beamlet with an initial phase φ_j at $z=0$ can be written as [13,18]

$$\begin{pmatrix} E_{jx}(x,y,0) \\ E_{jy}(x,y,0) \end{pmatrix} = \begin{pmatrix} F(x-x_j, y-y_j) PeG\left(x-x_j, y-y_j, z=0\right) \exp\left(i\phi_j\right) \\ 0 \end{pmatrix}, \tag{2}$$

where each beamlet carries the different initial phase $\varphi_j=2jm\pi/N$ in an orderly distribution and initial phase index m here is restricted to integer value. For simplicity, the off-axis positions of each beamlet and the embedded optical vortex are both set as $(x_j, y_j)=(\rho\cos\theta_j, \rho\sin\theta_j)$.

The PeG background beam in Eq. (2) is expressed as [13]

$$PeG(x,y,z=0) = \exp\left[-\frac{x^2+y^2}{w_0^2}\right] \times Pe\left(\frac{x}{x_0}, \frac{y}{y_0}\right) = \exp\left[-\frac{x^2+y^2}{w_0^2}\right] \times \int_{-\infty}^{\infty} \exp\left\{i\left[s^4+s^2\left(\frac{y}{y_0}\right)+s\left(\frac{x}{x_0}\right)\right]\right\} ds, \tag{3}$$

where $Pe(x/x_0, y/y_0)$ is the Pearcey function with scaling lengths x_0 and y_0 along x- and y-axes, respectively. To ensure the finite energy in real space, the Pearcey function is modulated by a Gaussian factor with waist width w_0.

The embedded vortex function $F(x-x_j, y-y_j)$ in Eq. (2) is described as [25]

$$F(x-x_j, y-y_j) = \left[x-x_j+i\,sgn(l)Q\left(y-y_j\right)\right]^{|l|}, \tag{4}$$

where l is the topological charge of the embedded optical vortex at $z=0$, $sgn(l)$ is sign function. The complex parameter Q is the noncanonical strength of the embedded optical vortex and for $Q=\pm1$ Eq. (4) is simplified to the canonical or symmetric vortex [25].

In the condition of coherent combination, the total electric field of the radial PeG vortex array beam at the $z=0$ plane can be expressed as

$$E_{coh}(x,y,0) = \sum_{j=0}^{N-1} E_{jx}(x,y,0). \tag{5}$$