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A theoretical study of the Fresnel diffraction of Laguerre-Bessel-Gaussian beam by a helical axicon

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ABSTRACT

Based on the Fresnel diffraction integral formula, the diffraction of Laguerre-Bessel-Gaussian (LBG) beam by a helical axicon is investigated. The expressions of the amplitude and the intensity distributions of the diffracted wave field are derived analytically by using the stationary phase method. Numerical examples are given to illustrate the variation of the intensity in radial and longitudinal directions versus the parameters of the incident LBG beam and the topological charge of the helical axicon. The results provide more general characteristics and diffraction by a helical axicon of Laguerre-Gaussian beam, Bessel-Gaussian beam and fundamental Gaussian beam are deduced as particular cases of this study.

1. Introduction

Nondiffracting optical beams are known to propagate indefinitely without change of the form and dimensions transverse intensity profile. In the literature, the first nondiffracting beams were introduced by Durnin in 1987 [1]. These beams are solutions of the free-space Helmholtz equation in cylindrical symmetry and have electric field amplitudes proportional to Bessel functions. The zeroth-order Bessel beam possesses a bright central core with sharp intensity, whereas higher-order Bessel beams have a dark central core surrounded by successive bright and dark rings. The Bessel beams can posses phase singularity on the optical axis and helical wave fronts which characterized them as vortex beams. They have potential applications in precision alignment, optical micromanipulation of particles, atom guiding and trapping [2,3]. Ideal Bessel beams are of infinite transverse extent and energy and thus cannot be realized experimentally. However, it is possible to generate finite extent approximation to Bessel beams which propagate over an extended distance in diffraction-free manner. A new solution of the paraxial wave equation has been introduced by Gori et al. [4], by modulating a Bessel function of zeroth order with a Gaussian profile, thus obtaining Bessel-Gaussian beams. Since then, the Bessel-Gaussian beams receive considerable interest by many optic researchers, such as [5–10]. The approximated zeroth-order Bessel beam was generated by using an annular aperture placed in the back focal plane of a lens, with a collimated beam [11], by using an axicon [12], and with computer generated holograms [13]. The possibility of generating a vortex nondiffracting higher order Bessel beam by use of optical diverging beam was firstly introduced by Vasara et al. in 1989 [14]. The authors showed that higher-order Bessel beam can be produced directly from an illuminating Gaussian beam by the use of axicon-type-computer generated holograms. Furthermore, Arlt et al. in Ref. [15] demonstrated a method for generating high order Bessel beams by illuminating

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an axicon with appropriate Laguerre-Gauss (LG) light beam. Recently, Topuzoski et al. [16,17] have studied the transformation by a helical axicon (HA) of an incident Laguerre-Gaussian beam into high order-Bessel beam with increased or reduced topological charge compared to the incident beam. More recently, Topuzoski [18] has investigated the generation of optical vortices with curved fork-shaped holograms for Gaussian laser beams. In addition, the theortical analysis of generalized spiraling Bessel beams with curved fork-shaped holograms using Laguerre-Gaussian, hypergeometric-Gaussian and Bessel-Gaussin beams, have been studied [19–21].

In this work, we propose a theoretical analysis of Fresnel diffraction by HA of the more general solution of the paraxial wave equation namely Laguerre-Bessel-Gaussian beam [22]. This mode is in the form of Laguerre polynomial, Bessel function, and Gaussian function. In the limit of large LG beam size, the Bessel factor dominates and LBG beam reduce to Bessel-like beam whereas in the opposite limit the mode reduces to LG beam. In the following, we present a theoretical analysis of the transformation by the HA of LBG beam with radial mode number n, azimuthal mode index l and the index q associated to the q^{th} order Bessel function J_q. The analytical expression of the intensity distribution of the diffracted wave field is derived by using the stationary phase method. The study provides more general expressions of diffracted paraxial wave field by the HA and many special cases are discussed. Some particular cases, which correspond to well known beams as LG beam, Bessel-Gauss beam and the fundamental Gaussian beam, are deduced. Numerical examples are also given to illustrate our analytical results.

2. Diffraction by a helical axicon of Laguerre-Bessel-Gaussian beams

The expression of the field distribution of the LBG mode at $z = 0$ reads [22]

$$U(r,\varphi,0) = C_{l,n}\left(\frac{r\sqrt{2}}{w_0}\right)^{l} L_n^l\left(\frac{2r^2}{w_0^2}\right) J_q\left(\mu\frac{r}{w_0}\right)\exp\left(-\frac{r^2}{w_0^2}\right)\exp\left[-i\left(l-q\right)\varphi\right], \tag{1}$$

where n and l are the radial and the azimuthal mode numbers, respectively, associated to the generalized Laguerre polynomial function. The parameter l is an integer which expresses the phase singularity with topological charge characterizing the beam as an optical vortex beam. q is the index associated to the q^{th} order Bessel function J_q, w_0 is the beam-waist radius of the Gaussian mode and μ/w_0 is the radial component of free space wave vector. (r, φ) denote the cylindrical coordinates and the coefficient $C_{l,n} = \frac{2}{\sqrt{1+\delta_{0,l}}}\cdot\sqrt{\frac{n!}{\pi(l+n)!}}$, where δ is the Kronecher symbol.

In the thin transparency approximation, the transmission function of a helical axicon (HA) is given by [15–17]

$$T(r,\varphi) = A(r)\exp\left(i\alpha r - ip\varphi\right), \tag{2}$$

where the axicon parameter $\alpha = k\,(n_r - 1)\,\gamma$ is connected to its internal angle γ, n_r is its refractive index, k denotes the wave number, $k = 2\pi/\lambda$, and λ is the wavelength. The integer p is the topological charge of the HA. The function $A(r)$ denotes the beam truncation function when the HA radius R_0 is smaller or equal to the transverse beam profile radius σ at the input plane $z = 0$, and it is defined as

$$A(r) = \begin{cases} 1, & \text{when } R_0 > \sigma, \\ circ\left(r/R_0\right), & \text{when } R_0 \le \sigma. \end{cases} \tag{3}$$

The diffraction of LBG beam by the HA which is situated at $z = 0$ can be described by means of the scalar diffraction theory. So, the diffracted wave field is found using the Fresnel-Kirchoff diffraction integral in the point $\left(\rho, \theta, z\right)$ [23]

$$\begin{aligned} U(\rho,\theta,z) &= \frac{ik}{2\pi z}.exp\left(-ik\left(z+\frac{\rho^2}{2z}\right)\right) \\ &\times\iint_{\Delta} T(r,\varphi)\,U(r,\varphi,0).exp\left(-i\frac{k}{2}\left(\frac{r^2}{z}-\frac{2r\rho\cos\left(\varphi-\theta\right)}{z}\right)\right) rdrd\varphi, \end{aligned} \tag{4}$$