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Cross-spectral purity of Stokes parameters, purity of partial polarization and statistical similarity

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ABSTRACT

In this paper, we discuss the concept of cross-spectral purity of Stokes parameters, purity of general partial polarization and implications of statistical similarity of an optical field. The conditions of cross-spectral purity of Stokes parameters and purity of partial polarization are obtained. Then, by utilizing statistical similarity, the relationships between the two purity and spatial coherence are explored. At last, a special situation that gives a relation of these two kinds of purity is briefly discussed.

1. Introduction

Since Young observed interference in the laboratory in 1801 [1,2], interference, as an important means of research in the development of optics, has been used for centuries. The classic Young's double-slit interference experiment has been widely used in the research of optics.

In Young's experiment, the spectral distribution of the light has been paid attention to. Generally, the normalized spectrum of the superposition will be different from the spectrum of fields at the pinholes. But in 1961, Mandel introduced the concept of cross-spectral purity of light beams [3]. When a region exists around a point in the plane of observation such that the spectral distribution of the light in this region is of the same form as the spectral distribution of the light at the pinholes, we say that the light at the pinholes be cross-spectrally pure. Recently, the concept has extended to the electromagnetic case [4], as well as to the cross-spectral purity of the Stokes parameters [5]. Cross-spectral purity and its influence on ghost imaging experiments were researched in [6]. The concept of statistical similarity which provide a new way to elucidate complete coherence has been introduced in [7]. The cross-spectrally pure field and statistical similarity were discussed in [8].

In the past few years, it has been found that the polarization state of light in interference experiment is also significant [9–15]. Recently Gori et al. [14] have studied the invariance properties of unpolarized light in Young's interferometer. The concept of polarization invariance in Young's interferometer was researched in detail by Santarsiero [12]. The purity of partial polarization was investigated by Hassinen [16], which is closely related to the cross-spectral purity.

In this paper, we introduce the cross-spectral purity of Stokes parameters firstly, and discuss relationship between statistical similarity and this cross-spectral purity. Next, the condition of purity of general partial polarization and the

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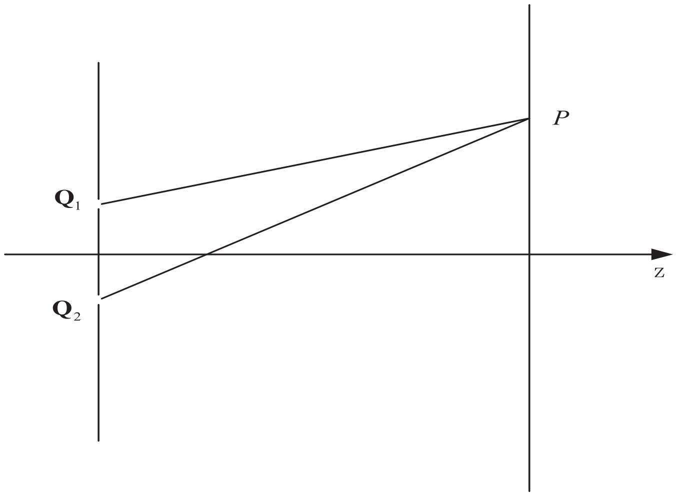

Fig. 1. Young's interference experiment.

implications of statistical similarity are also discussed. At last, we turn to a special situation when cross-spectral of Stokes parameters can ensure purity of partial polarization.

2. Cross-spectral purity of Stokes parameters

We start by considering the electromagnetic Young's interference experiment. We set a random, statistical stationary, electromagnetic beam $\mathbf{U}(\mathbf{r}, \omega)$, which denotes a statistical ensemble of the field variable of frequency ω at a point represented by a position vector $\mathbf{r}$. We define one of the initially the normalized Stokes parameters of the light at the pinholes, $\mathbf{Q}_1$ and $\mathbf{Q}_2$, to be the same at every frequency.

We assume that

$$s_j(\omega) = \frac{S_j(\mathbf{Q}_1, \omega)}{\int_0^\infty S_j(\mathbf{Q}_1, \omega)d\omega} = \frac{S_j(\mathbf{Q}_2, \omega)}{\int_0^\infty S_j(\mathbf{Q}_2, \omega)d\omega}, (j = 0, 1, 2, 3) \tag{1}$$

holds for one of the indices j = 0, 1, 2, 3. Here $S_j(\mathbf{r}, \omega)$ denote the Stokes parameters at point $\mathbf{r}$ .The measurements are made at point P on the observation plane which is parallel to the screen with pinholes (see Fig. 1). We can write for the particular Stokes parameters as:

$$S_j^{(1)} = \frac{|K_1|^2}{R_1^2} S_j(\mathbf{Q}_1, \omega), \tag{2a}$$

$$S_j^{(2)} = \frac{|K_2|^2}{R_2^2} S_j(\mathbf{Q}_2, \omega), \tag{2b}$$

where $S_j^{(n)}, (n = 1, 2)$ are the Stokes parameters in the observation plane when only the pinholes at $\mathbf{Q}_n$ is open, K_n are purely imaginary propagators, and R_n are the distances between the pinhole $\mathbf{Q}_n$ and the point $\mathbf{r}$. Then the Stokes parameters at the point on the observation plane, in view of the electromagnetic spectral interference law [10], can be written as

$$S_j(\mathbf{r}, \omega) = s_j(\omega) \left\{ \frac{|K_1|^2}{R_1^2} I_j(\mathbf{Q}_1) + \frac{|K_2|^2}{R_2^2} I_j(\mathbf{Q}_2) + 2\frac{|K_1||K_2|}{R_1 R_2} \sqrt{I_j(\mathbf{Q}_1) I_j(\mathbf{Q}_2)} \times \mathrm{Re}[\mu_j(\mathbf{Q}_1, \mathbf{Q}_2, \omega)\exp(-i\omega\tau_0)] \right\}, \tag{3}$$

where, $\tau_0 = (R_1 - R_2)/c$ is the time difference between light travelling from $\mathbf{Q}_1$ to $\mathbf{r}$ and $\mathbf{Q}_2$ to $\mathbf{r}$. And the intensity and normalization of the two-points Stokes parameters, $I_j(\mathbf{Q}_n)$ and $\mu_j(\mathbf{r}_1, \mathbf{r}_2, \omega)$, are defined as:

$$I_j(\mathbf{Q}_n) = \int_0^\infty S_j(\mathbf{Q}_n, \omega)d\omega, \tag{4a}$$

$$\mu_j(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{S_j(\mathbf{r}_1, \mathbf{r}_2, \omega)}{[S_j(\mathbf{r}_1, \omega)S_j(\mathbf{r}_2, \omega)]^{1/2}}, \tag{4b}$$