Optik 145 (2017) 336–345

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Analytical treatment of the couple stress fluid-filled thin elastic tubes☆

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ARTICLE INFO

Article history:
Received 10 November 2016
Received in revised form 19 May 2017
Accepted 23 May 2017

MSC:
00-01
99-00

Keywords:
Lie group symmetry
Self-adjointness
Soliton

ABSTRACT

In this paper, we present the symmetries and self-adjointness of the problem about the couple stress fluid-filled thin elastic tubes. Some soliton solutions of the specified problem are constructed with the aid of Lie group symmetry method.

1. Introduction

In most theoretical investigations on arterial pulse wave transmission through a thin elastic walled tubes, blood thickening due to rise in red blood cells has been assumed to be insignificant [1]. Many researchers have discussed and presented new models about flow in fluid-filled thin elastic tubes. Adesanya and co-workers have presented an investigation about the equations governing the fluid flow. They have used some assumptions and variable transformations to reduce the fluid flow equation to a new style of an evolution equation [2].

$$u_\tau + a_1 u u_\xi - a_2 u_{\xi\xi} + a_3 u_{\xi\xi\xi} + a_4 u_{\xi\xi\xi\xi} = 0. \tag{1}$$

Eq. (1) reduces to another type of evolution equations. When $a_2, a_3, a_4 \to 0$ Eq. (1) becomes the inviscid Burger's equation, $a_3, a_4 \to 0$ it becomes the viscous Burger's equation, $a_4 \to 0$ it is the KdV-Burger's equation and $a_3 = 0$, $a_2 = -1$ it becomes the Kuramoto–Sivashinsky (KS) equation.

Lie group symmetry method is one of the most powerful methods among the above mentioned methods, to determine exact solutions of NLPDEs in [3–15]. Furthermore, certain nonlinear PDEs admit infinitely many conservation laws. Although most lack a physical interpretation, these conservation laws play an important role and have many remarkable uses, specially in completely integrability. Some interesting papers in this field can be found in [16–24]. For more details see [48–57].

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http://dx.doi.org/10.1016/j.ijleo.2017.05.082
0030-4026/

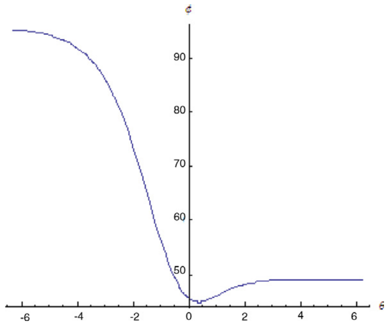

Fig. 1. 2D figure of the solution of Eq. (1) with aid the reduced Eq. (7).

2. Symmetries of Eq. (1)

In here, we give a discussion through the symmetries of the problem.

Firstly, we give Lie group of infinitesimal transformations:

$$\tau^* = \tau + \epsilon\zeta^1(\tau, \xi, u) + O(\epsilon^2),\ \xi^* = \xi + \epsilon\zeta^2(\tau, \xi, u) + O(\epsilon^2),\ u^* = u + \epsilon\phi(\tau, \xi, u) + O(\epsilon^2), \tag{2}$$

where ϵ is the group parameter. The associated Lie algebra of infinitesimal symmetries is the set of the vector field of the form

$$X = \zeta^1(\tau, \xi, u)\frac{\partial}{\partial\tau} + \zeta^2(\tau, \xi, u)\frac{\partial}{\partial\xi} + \phi(\tau, \xi, u)\frac{\partial}{\partial u}. \tag{3}$$

If $Pr^{(4)}X$ denotes the fourth prolongation of X then the invariance condition is

$$Pr^{(4)}X(\Delta)|_{\Delta=0} = 0, \tag{4}$$

where $\Delta := u_\tau + a_1uu_\xi - a_2u_{\xi\xi} + a_3u_{\xi\xi\xi} + a_4u_{\xi\xi\xi\xi}$, and yields an over-determined system of linear PDEs in ζ^1, ζ^2 and ϕ, the so-called determining equations, which solving these equations in different cases, we get:

Case 1: $a_1a_4 \neq 0$, $a_3 = 0$, $a_2 = -1$.

This case is related to the KS equation and we have

$$\zeta^1 = C_1,\ \zeta^2 = a_1\tau C_2 + C_3,\ \phi = C_2, \tag{5}$$

where C_1, C_2 and C_3 are arbitrary constants. Thus the Lie symmetry algebra admitted by Eq. (1) is spanned by the following three infinitesimal generators (Fig. 1)

$$X_1 = \frac{\partial}{\partial\tau},\ X_2 = \frac{\partial}{\partial\xi},\ X_3 = a_1\tau\frac{\partial}{\partial\xi} + \frac{\partial}{\partial u}, \tag{6}$$

We present below a reduction and related solution with some different generators:

Reduction 1.1. Similarity variables related to $X_1 + \nu X_2$ are $u(\tau, \xi) = \Phi(\theta)$, where $\theta = \xi - \nu\tau$ and satisfies the following equation:

$$(a_1\Phi(\theta) - \nu)\Phi'(\theta) + \Phi''(\theta) + a_4\Phi^{(4)}(\theta) = 0. \tag{7}$$

Eq. (7) has the following soliton solutions

$$\Phi(\theta) = \frac{1}{19a_1}\left[30 + 19\nu - \frac{3(30 + 19\nu)}{2(1 + \cosh\theta + \sinh\theta)^2} + \frac{120}{(1 + \cosh\theta + \sinh\theta)^3}\right]$$

Case 2: $a_1a_2a_3a_4 \neq 0$.

This case is more general one and we can get

$$\zeta^1 = C_2,\ \zeta^2 = a_1\tau C_1 + C_3,\ \phi = C_1, \tag{8}$$

where C_1, C_2 and C_3 are arbitrary constants. Thus the Lie symmetry algebra admitted by Eq. (1) is spanned by the following three infinitesimal generators

$$X_1 = \frac{\partial}{\partial\tau},\ X_2 = \frac{\partial}{\partial\xi},\ X_3 = a_1\tau\frac{\partial}{\partial\xi} + \frac{\partial}{\partial u}, \tag{9}$$

We present a reduction and related solutions with some different generators: