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Nonparaxial propagation of a partially coherent four-petal Gaussian beam in free space

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ABSTRACT

Based on the generalized Rayleigh-Sommerfeld diffraction integral, the equations of non-paraxial propagation, far field propagation and paraxial propagation of a partially coherent four-petal Gaussian beam in free space are obtained. The results show that the far field propagation and paraxial propagation can be regarded as special cases of nonparaxial propagation. The normalized intensity and degree of coherence of a nonparaxial partially coherent four-petal Gaussian beam are illustrated and analyzed using numerical example.

1. Introduction

In past years, the nonparaxial propagation of laser beam in free space has attracted much attention when the beam width and wavelength of laser beam are comparable. In this situation, the paraxial propagation will become invalid. Until now, based on the generalized Rayleigh-Sommmerfeld diffraction integral, the nonparaxial properties of various beams propagating in free space have been widely illustrated, such as those of spirally polarized optical beam [1], elliptical Gaussian beam [2], radially polarized light beam [3], four-petal Gaussian beam [4], hollow Gaussian beam [5], partially coherent dark hollow beam [6], rotating Cosh-Gaussian beam [7], flat-topped vortex hollow beam [8,9], Gaussian optical vortex beam [10], partially coherent flat-topped beam [11], partially coherent anomalous hollow beam [12], Lorentz-Gauss beam [13,14], multi-Gaussian Schell-model beam[15,16].

On the other hand, with the development of laser technology, new laser beam has also been introduced. Recently a new laser beam called four-petal Gaussian beam has been produced and studied [17]. Based on the model of four-petal Gaussian beam, the propagation properties of laser beams based on four-petal Gaussian beam propagation in free space, optical system, uniaxial crystal and turbulent atmosphere have been wide studied [18–22]. However, to the best of our knowledge, there has no reports about the nonparaxial propagation of a partially coherent four-petal Gaussian beam in free space. In this paper, we have studied the nonparaxial propagation of a partially coherent four-petal Gaussian beam in free space using the generalized Rayleigh-Sommmerfeld diffraction integral. The intensity and coherence properties of a partially coherent four-petal Gaussian beam in free space have been illustrated and analyzed.

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2. Theory analysis

Now, in the Cartesian coordinate system, the cross-spectral density function of a partially coherent four-petal Gaussian beam generated by a Schell-model source at the source plane can be expressed as [23]:

$$W(\mathbf{r}_{10}, \mathbf{r}_{20}, 0) = \left(\frac{x_{10}y_{10}}{w_0^2}\right)^{2N} \left(\frac{x_{20}y_{20}}{w_0^2}\right)^{2N} \exp\left(-\frac{r_{10}^2 + r_{20}^2}{w_0^2}\right) \times \exp\left[-\frac{(x_{10}-x_{20})^2}{\sigma_x^2} - \frac{(y_{10}-y_{20})^2}{\sigma_y^2}\right] \tag{1}$$

where N represents the order of four-petal Gaussian beam; w_0 denotes the waist width of Gaussian beam; σ_x and σ_x are the transversal coherence length of partially coherent beam. $\mathbf{r}_{10} = (x_{10}, y_{10})$ and $\mathbf{r}_{20} = (x_{20}, y_{20})$ are the position vectors at the source plane.

According to the generalized Rayleigh-Sommerfeld diffraction integral, the cross-spectral density function of partially coherent beams propagating in free space at the receive plane z can be expressed as:

$$W(\mathbf{r}_1, \mathbf{r}_2, z) = \left(\frac{z}{\lambda}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\mathbf{r}_{10} d\mathbf{r}_{20} W(\mathbf{r}_{10}, \mathbf{r}_{20}, 0) \frac{\exp[ik(R_2 - R_1)]}{R_1^2 R_2^2} \tag{2}$$

where $\mathbf{r}_1 = (x_1, y_1)$ and $\mathbf{r}_{10} = (x_1, y_1)$ are the position vectors at the receiver plane z; $k = 2\pi/\lambda$ is the wave number; λ is wavelength. R_1 and R_2 of Eq. (2) are expressed as:

$$R_1 = \sqrt{(x_1 - x_{10})^2 + (y_1 - y_{10})^2 + z^2} \tag{3a}$$

$$R_2 = \sqrt{(x_2 - x_{20})^2 + (y_2 - y_{20})^2 + z^2} \tag{3b}$$

To investigate the propagation properties of partially coherent four-petal Gaussian beam beyond the paraxial approximation, R_1 and R_2 in Eq. (2) can be expanded as:

$$R_1 = r_1 + \frac{x_{10}^2 + y_{10}^2 - 2x_1x_{10} - 2y_1y_{10}}{2r_1} \tag{4a}$$

$$R_2 = r_2 + \frac{x_{20}^2 + y_{20}^2 - 2x_2x_{20} - 2y_2y_{20}}{2r_2} \tag{4b}$$

where

$$r_1 = \sqrt{x_1^2 + y_1^2 + z^2} \tag{5a}$$

$$r_2 = \sqrt{x_2^2 + y_2^2 + z^2} \tag{5b}$$

Recalling the following formulae [24]

$$\int_{-\infty}^{+\infty} x^n \exp\left(-px^2 + 2qx\right) dx = n! \exp\left(\frac{q^2}{p}\right) \left(\frac{q}{p}\right)^n \sqrt{\frac{\pi}{p}} \sum_{k=0}^{[\frac{n}{2}]} \frac{1}{k!(n-2k)!} \left(\frac{p}{4q^2}\right)^k \tag{6}$$

$$H_n(l) = \sum_{k=0}^{[\frac{n}{2}]} \frac{(-1)^k n!}{k!(n-2k)!} (2l)^{n-2k} \tag{7}$$

And substituting Eqs. (1) and (4) into Eq. (2), and replacing R_1 and R_2 of Eq. (2) in the dominators with r_1 and r_2, we can obtain

$$W_{np}(\mathbf{r}_1, \mathbf{r}_2, z) = \left(\frac{z}{\lambda}\right)^2 \left(\frac{1}{w_0^2}\right)^{4N} \frac{\exp[ik(r_2 - r_1)]}{r_1^2 r_2^2} W_{np}(x, z) W_{np}(y, z) \tag{8}$$