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# Probability distribution of random cylinder's total scattering cross section

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ABSTRACT

An arbitrary shaped scatterer is often modeled as a canonical structure while solving an electromagnetic boundary value problem. This approach approximates the boundary to provide complex analytical solutions. However, most real life structures do not have canonical or regular boundaries. Determining an analytical solution of electromagnetic scattering problem is quite a tedious task, especially for random geometries. This work proposes a method to compute the scattering statistics of a random cylinder. A formulation is developed to determine the probability density function (pdf) of scattering cross section, $\sigma_T$, from the statistics of random cylinder's radius. This approach eliminates the need to actually solve an electromagnetic scattering problem. The optical theorem is employed to estimate the pdf of $\sigma_T$ from the pdf of lateral width. The proposed results are found in good agreement with the numerically simulated results.



## 1. Introduction

Scattering of electromagnetic waves has been a widely discussed research area for scientists. Recently, scattering from random objects has also attracted the attention of research community [1–7]. Various techniques are developed to study scattering from planar random/rough surfaces [8–11]. Relatively little work is found in literature on 2D cylindrical objects with random cross sections [12–15]. The randomness of the scattering surface makes it difficult to apply exact analytic techniques. Approximate analytical or numerical techniques are employed to determine scattering parameters of random/rough scatterers. In most of the engineering problems, it is desirable to have an estimate of a parameter rather than its exact value. We propose a method to estimate the statistics of $\sigma_T$, without actually solving the electromagnetic scattering problem, of random cylinder.

A random cylinder is a 2-D surface, $\mathbf{r}(\theta)$ and is independent of $z$ coordinates. The boundary curve $\mathbf{r}(\theta)$ is referred to as the radius of random cylinder, with a probability density function (pdf). Total scattering cross section ($\sigma_T$) is the total power scattered by a perfect electrically conducting (PEC) scatterer. It depends on the cross sectional shape of cylindrical scatterer [16]. Optical theorem relates $\sigma_T$ of an object to its lateral width. In the optical frequency limit, the $\sigma_T$ of a PEC cylindrical object is equal to twice its lateral width. Lateral width is maximum projection of a cylindrical structure on a plane normal to the direction of incident. It is a geometrical parameter and its pdf can be obtained from the radius of random cylinder. Therefore, an estimate of the pdf of $\sigma_T$ can be obtained from the pdf of lateral width. The formulation obtained is applied to Gaussian random cylinder. It is shown that the estimated and simulated results agree closely.

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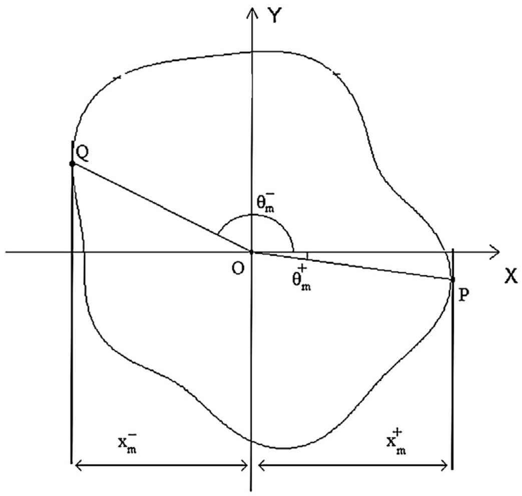


**Fig. 1.** A random cylinder and its cross sectional width.

The rest of this paper is organized as follows: Section 2 includes a detailed formulation of the problem addressed. Section 3 provides numerical simulations and results. Finally, the conclusions are presented in Section 4. Throughout the text, the bold-face letters represent the random variables.

## 2. Formulation

Consider Fig. 1, in which a plane wave propagating along $y$-axis is incident on a random cylinder. The axis of the cylinder is along $z$-axis, and the maximum projection of the random cylinder on the normal plane is the lateral width of the random cylinder. Any $y$ = constant plane is a normal plane in this configuration.

According to the figure, $\mathrm{x}_\mathrm{m}^+$ and $\mathrm{x}_\mathrm{m}^-$ are the maximum projection points on the normal plane. These are the projections of points $P$ and $Q$ of the random cylinder, respectively. Let $\theta_m^+$ and $\theta_m^-$ are the angles which position vectors $P$ and $Q$ form with positive $x$-axis, respectively. Considering the fact that center of the cylindrical object is taken to be at origin, the lateral width ($\mathbf{l_w}$) can be written as,

$$\mathbf{l_w} = \mathbf{x_m^+} + |\mathrm{x}_\mathrm{m}^-| \tag{1}$$

$\mathrm{x}_\mathrm{m}^+$ and $|\mathrm{x}_\mathrm{m}^-|$ are independent and identically distributed (i.i.d) random variables. The pdf of maximum projection of the radius on positive $x$-axis, $\mathrm{x}_\mathrm{m}^+$, is determined first. Let $\mathbf{x}^+(\theta)$ be the projection of radius on positive $x$-axis, given by:

$$\mathrm{x}^+(\theta) = \mathbf{r}(\theta)\cos\theta, \ -\pi/2 < \theta < \pi/2, \tag{2}$$

where, $\mathbf{r}(\theta)$ is random radius. The pdf of $\mathbf{x}^+$ can be written as a scaled pdf of $\mathbf{r}(\theta)$, $f_\mathbf{r}(r)$, as in [11]:

$$f_{\mathrm{x}^+}(x) = \frac{1}{|\cos\theta|} \quad f_\mathbf{r}\left(\frac{x}{\cos\theta}\right) \tag{3}$$

In general, the random process $\mathbf{x}^+(\theta)$ is a non stationary random process. Maximum of the process $\mathbf{x}^+(\theta)$ for $-\pi/2 < \theta < \pi/2$ is the maximum projection of radius on positive $x$-axis, given as,

$$\mathrm{x}_\mathrm{m}^+ = \max_{-\frac{\pi}{2}<\theta<\frac{\pi}{2}} [\mathrm{x}^+(\theta)]. \tag{4}$$

A system which determines the maximum is a non linear system, has memory and its inverse system does not exist. These properties of max(·) function together with non-stationarity of the input process, $\mathbf{x}^+$ makes it difficult to analytically determine the statistics of $\mathrm{x}_\mathrm{m}^+$ from the statistics of $\mathbf{x}^+$. Therefore, numerical techniques are employed to estimate the pdf of $\mathrm{x}_\mathrm{m}^+$.

If the power spectral density of the radius is band limited, then power spectral density of its projection will also be band limited. According to the stochastic sampling theorem, a band limited process can be fully represented by its samples [17]. Hence, $K$ equally spaced samples of $\mathbf{x}^+$ are obtained. The maximum projection, $\mathrm{x}_\mathrm{m}^+$ is therefore

$$\mathrm{x}_\mathrm{m}^+ = \max(\mathrm{x}_\mathbf{1}^+, \mathrm{x}_\mathbf{2}^+, \ldots, \mathrm{x}_\mathrm{K}^+). \tag{5}$$

and its Cumulative distribution function can be written as [18]:

$$F_{\mathrm{x}_\mathrm{m}^+}(x) = \Pr(\mathrm{x}_\mathrm{i}^+ \le x_m^+, \forall \quad i) = F_{\mathrm{x}^+}(x_m^+, x_m^+, \ldots, x_m^+) \tag{6}$$