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Bursting behavior in degenerate optical parametric oscillator under noise

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ABSTRACT

The distribution of Lyapunov exponents is calculated in the two-parameter region, and parameter is set carefully to reproduce bursting state from the optical parametric oscillator thus can be consistent with the previous experiments. The input field amplitude is changed to trigger different modes of outputs and then external noise is imposed to detect the transition of subharmonic mode and fundamental mode, it could be helpful to understand the potential mechanism for electromagnetic radiation-induced transition of electrical activities in neurons.

1. Introduction

Chaos can be detected and observed in physical, chemical and even biological systems [1–8]. Oscillators and maps can be helpful to reproduce chaotic behaviors in sampled times from theses dynamical models. Within the topics of chaos and hyperchaos, many schemes have been proposed to suppress chaos, modulation of dynamics and realize synchronization between chaotic systems [9–17]. On the other hand, generating different frequency and/or modes in outputs from these chaotic systems could be useful in signal processing by using appropriate control schemes. In numerical and theoretical studies, periodical oscillators [18] are helpful to investigate the oscillating behaviors of some complex systems, and even can model the nonlinear properties. It is known that Hodgkin-Huxley neuron model [19] and its simplified versions can be effective to reproduce the electrical activities in neurons, and these models [20–24] have been extensively used for computational neuroscience. In fact, some realistic factors should be considered while these neuron models prefer to produce similar electrical modes in neuron. For example, the effect of electromagnetic radiation [25,26] on neuron should be considered. Some researchers thought magnetic flux [27,28] could be used to model the effect of electromagnetic induction and radiation on neurons, and this model can also be effective to explain the emergence of multiple modes in electrical activities. However, what is the difference when light is imposed on neuron? It is believed that light belongs to electromagnetic wave and can change the excitability of neurons as well. Indeed, it needs model resetting to include the effect of light when neurons are exposed to laser irradiation.

In fact, neurons are polarized and magnetized when external electromagnetic field is considered. The degenerate optical parametric oscillator (DOPO) [29–34] can detect the response to external field, and the outputs can generate complex dynamical properties. Particularly, it is found in experiments that DOPO can generate bursting behaviors [35]. As a result, it is interesting to find appropriate parameter regions so that different modes of oscillating behaviors can be selected. In this paper, based on the DOPO model, the zero Lyapnuov exponents [36] region is explored to find bursting behaviors from the

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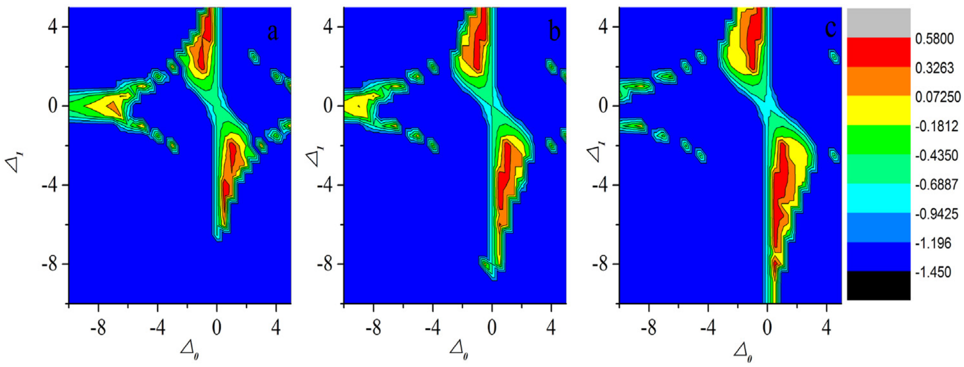

Fig. 1. Distribution for largest Lyapunov exponents in detuning parameter region, the external input field is fixed for (a)$E_A = 7$; (b) $E_A = 9$; (c) $E_A = 12$. The parameter is set as $\gamma = 1$, and the snapshots are plotted in color scale.

sampled time series by applying different field intensity. Finally, the noise is also considered to discern the mode transition of bursting behaviors. It is found that DOPO model can produce multiple modes in outputs, therefore, it could be helpful for further model setting of neurons exposed to laser irradiation.

2. Model description

The local kinetics for the DOPO is described by complex dynamical equations as follows

$$\begin{cases} \dot{A}_1 = -(1 + i\Delta_1)A_1 + A_1^* A_0 \\ \dot{A}_0 = -(\gamma + i\Delta_0)A_0 + E_A - A_1^2 \end{cases} \tag{1}$$

where A_1, A_0 in Eq. (1) describes the complex amplitude of the subharmonic mode and fundamental mode, respectively. The parameter γ denotes the reduced decay rate of the fundamental model, Δ_1 and Δ_0 are detuning parameters. E_A often describes the input field amplitude and it is often chosen with some positive values. For simplicity, the complex equations are reproduced by a four-variable dynamical equations by setting $A_1 = x + iy$, $A_0 = z + iw$, it reads,

$$\begin{cases} \dot{x} = -x + \Delta_1 y + xz + yw \\ \dot{y} = -\Delta_1 x - y - yz + xw \\ \dot{z} = -\gamma z + \Delta_0 w - x^2 + y^2 + E_A \\ \dot{w} = -\gamma w - \Delta_0 z - 2xy \end{cases} \tag{2}$$

That is, the four-variable equations are in Eq. (2) are equivalent with the description for the complex variable equations in Eq. (1), the external field E_A will be changed to different intensities at different detuning parameter regions, also, Gaussian white noise will be considered, the dynamical responses in the outputs will be discussed. For simplicity, it reads,

$$I = |A_1^2 + A_0^2| \tag{3}$$

3. Numerical results and discussion

In the numerical calculation, the fourth-order Rung-Kutta algorithm is carried with time step $h = 0.01$, the initials are selected as (1, 1, 1, 1). The spectrum of Lyapunov exponents is calculated by using the Wolf algorithm. According to Eq. (2), the intensity of external input field amplitude is changed to detect the largest Lyapnuov exponent in the two-parameter region for Δ_1 and Δ_0 (Fig. 1).

As is known, emergence of chaotic behavior means that positive Lyapunov exponent at least is detected, while stability is reached when the largest Lyapunov exponent becomes negative. Indeed, oscillating state could be approached when the largest Lyapunov exponent is much close to zero. According to the distribution for largest Lyapunov exponents, it is found that zero Lyapunov exponent can be detected under appropriate detuning parameter setting and chaotic behavior can be enhanced by furthering increasing the intensity of external field. To discern the rhythm in periodical state, the interspike interval (ISI) is calculated at fixed parameter $\gamma = 1$, $\Delta_0 = 0.5$, $\Delta_1 = -7$. Furthermore, the bifurcation analysis is carried out by setting different external field intensity, and the results are plotted in Fig. 2.

It is found that the oscillator becomes death when the external field is below $E_A = 8.2$, then the system begins trigger multiple periodical modes in the outputs and could be much similar to the electrical properties in neurons subjected to external forcing current. In Fig. 3, sampled time series are calculated to discuss the rhythm in the outputs.