Optik 139 (2017) 272–282

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

A note on applications of time-domain solution of Cole permittivity models

Baris Baykant Alagoz [a,*], Gulizar Alisoy [b], Serkan Alagoz [c], Hafiz Alisoy [d]

[a] Department of Computer Engineering, Inonu University, Malatya, Turkey
[b] Department of Mathematics, Namik Kemal University, Tekirdag, Turkey
[c] Department of Physics, Inonu University, Malatya, Turkey
[d] Department of Electronics and Communication Engineering, Namik Kemal University, Tekirdag, Turkey

ARTICLE INFO

Article history:
Received 4 January 2017
Accepted 3 April 2017

Keywords:
Dielectric permittivity
Cole-Cole model
Davidson-Cole model
Fractional order dynamic capacitance modeling
Electric field flux density
Wave impedance

ABSTRACT

The complex dielectric permittivity is an important parameter for characterization of electrical properties of dielectric materials. Experimental studies demonstrated that Cole models of dielectric permittivity can provide a better fitting to the experimental data because of allowing for fractional-order frequency dependence. This study aims to investigate physical interpretation of time domain solutions of Cole permittivity models. For this purpose, impulse responses of Cole-Cole model and Davidson-Cole model are expressed in Mittag-Leffler function form by using inverse Laplace transform. The impulse responses of these models are decomposed into impulsive and dispersive components, and the relations of these components with relaxation mechanism of dielectric materials are discussed. By considering impulse response solution of Cole-Cole models, a fractional order dynamic capacitance model is introduced for time domain equivalent circuit modeling of dielectric materials. Moreover, transient properties of electromagnetic wave penetration to dielectric materials are analyzed according to impulse response solution of Cole-Cole model. To illustrate applications of proposed time domain permittivity solutions, the Cole-Cole model of ethyl-acetate liquids was also studied and results are presented.

1. Introduction

One of the substantial parameters determining electromagnetic wave propagation properties in dispersive mediums is the dielectric permittivity. The complex dielectric permittivity has been commonly used for the characterization of dielectric materials in material science and related technologies. The permittivity of dielectric materials strongly depends on the frequency of electric field disturbances because various mechanisms responding the field disturbance are excited within the different frequency regions [1]. While an electric field disturbance is penetrating through the dielectric materials, the charged particles (space charges, free radical, molecular dipols and atoms) in the dielectric body are enforced to rearrange themselves in order to line up dipole moments [1]. This rearrangement of charged particles allows increasing the electrical flux density in the dielectric material structure and thus the electrical field penetration in the material can take place. This rearrangement phenomenon, increasing electrical field flux density, takes some time depending on movement capability of charged particles in the structure, and it is commonly referred to as relaxation time and the process is called dielectric

* Corresponding author.
E-mail address: baykant.alagoz@inonu.edu.tr (B.B. Alagoz).

http://dx.doi.org/10.1016/j.ijleo.2017.04.010

relaxation of the materials. The relaxation time is also one of the important characteristic properties of dispersive materials, which determines the frequency-dependent features of the dielectric mediums.

Complex charge structures of polar dielectric material body lead to material response strongly frequency-dependent. Alternating field disturbances activate the different mechanisms of polar dielectrics at different frequency ranges. For instance, the field disturbances in low and mid-frequency regions (10 m Hz–10 GHz) mostly affect the motion of space charges and molecular dipoles in the dielectric structure, and lead to the molecular polarization in these frequency ranges. The relaxation mechanisms of dielectrics are mainly effective in these frequency ranges. At the much higher frequencies, resonance mechanisms of atoms and valence electrons mainly take effect on the atomic system [1].

Analysis of complex dielectric permittivity, based on the time response of materials, has technological implications in many fields such as material science [2], biophysics [3], electromagnetics [4], bio-impedance measurements [5], geophysics [6,7]. Specifically, the time-domain reflectometry (TDR) [8] employs the short-duration electromagnetic pulses to estimate the frequency-dependent dielectric properties of materials by considering the pulse responses. Fundamentally, the dynamic response of dielectric materials and their characteristic relaxation times are observable via the time (transient) response of dielectric materials. Nowadays, complex permittivity models considering relaxation mechanisms are very essential for the characterization of electrical properties of complex material structure, particularly in biophysics and geophysics studies [9,10].

For the frequency domain modeling of dielectric materials, two useful models have a wide acceptance in the material science. These are Debye model and Cole-Cole model. Debye model is used to characterize integer-order dynamics of dielectric permittivity by allowing only integer order angular frequency dependence. It is indeed a physically derived model of dipolar relaxation [11,12] and considers the relaxation response of an ideal, non-interacting population of dipoles to an alternating electric field. In fact, Cole-Cole model emerged as a very useful modification of the Debye model, which allows fractional-order of angular frequency, and thus it presents the potential of providing a better accuracy in matching experimental data [13]. But, as a result of the fractional-order nature of Cole-Cole model, the time domain solutions of Cole-Cole model is more complicated compared to the time domain solutions of Debye model. Since the fractional-order frequency dependency, the time domain solution of Cole-Cole model can be expressed arithmetically by time series with infinite terms and therefore it can be calculated approximately in practice [13]. To address the numerical solution of fractional order partial differential equations of specific physical systems, approximate solution methods are also proposed [13,14]. On the other hand, despite the fact that it can provide better fitting to experimental measurements, the physical interpretation of Cole-Cole model is not easy as it is for Debye model [13]. In fact, there is not a common understanding for the physical interpretation of fractional order derivative operator. Preferably, the fractional order derivative operators are gaining their physical meaning according to appearing dynamics of modeled systems. This paper also aims to discuss the physical interpretation of time domain solutions of Cole-Cole complex permittivity model in application of the material response to the field disturbances.

In historical progress of Cole models, an alternative model, which is also including fractional order dynamics, has been proposed by Davidson and Cole for resembling Debye medium in the low-frequency limits, and it has been referred to as Davidson-Cole model [15]. Davidson-Cole model is preferred, particularly when the dielectric loss peak shows asymmetric broadening.

In practice, the time and frequency response measurements of complex dielectric permittivity were commonly utilized for the extraction of Debye or Cole-Cole models of materials [7,10,16,17]. In general, after obtaining relevant model parameters from measurements, Debye or Cole-Cole models of materials can be identified by considering experimental data [6,12].

In this study, we benefit from the recent advances in engineering solutions of fractional calculus in order to calculate impulse response of Cole models. To obtain a transfer function regarding to the relaxation response of dielectric materials under an alternating electrical field excitation, Cole models of the complex dielectric permittivity are expressed in the s-domain by applying $j\omega = s$ transformation. Then, the transfer functions of the Cole models are solved by inverse Laplace transform techniques. The current paper addresses time response analyses of bipolar dielectric materials according to the impulse response of Cole models. The effects of model parameters and their relations with relaxation mechanisms are discussed briefly. We also constitute relations of these models with Maxwell equation and explain transient properties of electromagnetic wave penetration to dielectric material on the bases of the impulse response solutions of Cole models. Moreover, a dynamic capacitance model is developed according to time response of Cole-Cole permittivity model. To present an illustrative study, we analyzed the experimental Cole-Cole model of the ethyl-acetate liquid and the results of analyses are discussed.

2. Methodology

2.1. Fractional-order transfer function of Cole-Cole and Davidson-Cole models of the complex dielectric permittivity

The response of dielectric materials to the static electric field is defined as $\varepsilon_s = \lim_{\omega\to 0}\varepsilon(\omega)$, which is also known as the static permittivity. At the high-frequencies, the permittivity of the material approximates to $\varepsilon_\infty = \lim_{\omega\to\infty}\varepsilon(\omega)$. Here, $\varepsilon_s \in R$ and $\varepsilon_\infty \in$ R are constants. The Cole-Cole model of the complex dielectric permittivity was given depending on the angular frequency (ω) and the characteristic relaxation time (τ) of the material as follows [8,15],