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Hopf bifurcation and chaos in a fractional order delayed memristor-based chaotic circuit system

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ABSTRACT

This paper present Hopf bifurcation and chaos in a fractional order delayed memristor-based chaotic circuit system. Firstly, regarding the time delay τ as a bifurcation parameter, we investigate the stability and bifurcation behaviors of this fractional order delayed memristor-based chaotic circuit system. Some explicit conditions for describing the stability interval and emergence of Hopf bifurcation are derived. Secondly, corresponding to different system parameters, the complex dynamics behaviors of this system are discussed by using the bifurcation diagrams, the Max Lyapunov exponents (MLEs) diagram, the time domain waveforms, the phase portraits and the power spectrums. Thirdly, we study the influence of the two parameters (time delay τ and fractional order q) on the chaotic behavior, and it is found when time delay τ and fractional order q increases, the transitions from period one to period two and period four to chaos are observed in this memristor–based system. Meanwhile, corresponding critical values of time delay τ and fractional order q, the lowest orders q and the minimum time delay τ for generating chaos in the fractional order delayed memristor–based system are determined, respectively. Also, when the system occurs period one, the corresponding frequency is verified theoretically and experimentally. Finally, numerical simulations are provided to demonstrate the validity of theoretical analysis using the improved Adams–Bashforth–Moulton algorithm.

1. Introduction

In 1971, according to the completeness of axiomatic system in circuit theory, Professor Leon O. Chua defined the relationship between the magnetic flux and charge, and he put forward the concept of a memristor which was a fourth basic circuit components (the resistor, inductor, capacitor) [1–5]. In May 2008, the Stanley Williams team of HP Labs successfully developed a memristor physical model, which can work on confirming the existence of the memristor [6]. Since then, the memristor has been development rapidly, which has aroused widespread concern in the academia and industry. Because of a nonlinear characteristic in the memristor, it has great potential application such as reconfigurable logic and programmable logic devices, neural network [7–9], signal processing [10,11], non-volatile memory [12] and non-linear circuit [13–16] and so on. Accordingly, with the increasing depth research of the theoretical model and device performance in memristor, a variety of memristor mathematical models have been proposed and used in various applications. Also, using the memristor in secure communications is a relatively important area in the field of artificial intelligence, which attracts many researchers

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to work. This is a very wide range of applications that was used to build a memristor circuit to generate non-linear chaotic signal to the field in secure communications.

Fractional calculus is an extension and generalization of the integer order calculus to any real order of calculus. It has more than 300 years of history [17,18]. Because of no real prospect of the theory in fractional calculus a long time, it has not developed slowly. Comparing to an integer-order differential equations, fractional differential equations can more accurately describe many natural phenomena in many fields of applied science, such as electromagnetic waves, polarization electrolyte viscoelastic system, encryption [19], biology, system control [20–22], signal processing [23] and so on. However, it is the first time that the Mandelbrot pointed out there are a lot of fractal dimension theory in many fields of science and technology in 1983 [24]. Then, the application of fractional calculus is caused widespread concern, and gradually become a hot research topic of nonlinear theory. Fractional order system study is based to integer order system study, that fractional differential operator is introduced into the nonlinear dynamic system in the integer-order system. Then, many more classic dynamics behaviors of fractional order system are analyzed, such as Chua's circuit [25], Duffing system [26], Chen system [27] and so on.

Then, it is inevitable that the existence of time delay in many systems. Although the time delay is very small, it often affects the dynamic behavior of the whole system. Therefore, the delayed fractional order system must be considered to study. Therefore, a large number of researchers studied the fractional order delayed system, such as economy [28], biology [29], system control [30–32] and so on.

As far as we know, there are many investigations on the complex dynamics analysis of bifurcation and chaos in the fractional order delayed system [33–36]. In [33], the fractional order delayed predator-prey systems with Holling type-*II* functional response was analyzed, and Hopf bifurcation occured when time delay τ passed through a sequence of critical values. In [34], the bifurcation analysis of a fractional order single cell with delay which was proposed for delayed cellular neural networks with respect to the time delay τ. In [35], the dynamic of a tri-neuron fractional neural network was investigated, and applying the sum of time delay as the bifurcation parameter was studied. Also, Dumitru et al. [36] analyzed chaotic behavior in the fractional order nonlinear Bloch equation with delay. However, those delayed system is only study one of the bifurcation analysis and chaos, and there are few results with the respect to the system that is analyzed of the dynamic in Hopf bifurcation and chaos behavior.

Till now, there are few results with the Hopf bifurcation and chaos behavior in a fractional order delayed memristor-based chaotic systems. Donato et al. [37] was apply the integer order memristor-based chaotic system to the fractional order system, and chaotic behaviors was shown. Lin et al. [38] also analyzed the chaotic behavior in a fractional order memristor-based chaotic circuit using the fourth degree polynomial. MS et al. [39] analyzed the Hopf bifurcation and chaos in a fractional order memristor-based electrical circuit in system parameter and fractional order q. However, those memristor-based chaotic systems are all not considered the exist of time delay τ. So it is necessary and challenging that we study two parameters of time delay τ and fractional order q to influence complex dynamic behaviors in the memristor-based circuit system.

Through the above discussion, this paper presents a system based on delayed memristor circuit to analysis the Hopf bifurcation and chaotic behavior. The main contribution of this paper is to present the influence and relation of the two parameters of the fractional order q and the time delay τ on the dynamic behavior of the system. Accordingly, nonlinear dynamic is illustrated using time-domain diagram, phase diagram, spectrum, bifurcation diagram, Max Lyapunov exponents. Simulation results show that the fractional order delayed systems can generate complex nonlinear dynamic behavior with lower order. Also, the bifurcation point of time delay τ^* and fractional order q^* is to verify correctly in the theoretical value and the critical frequency in period one is verified accurately.

This article follows: The Section 2 describes the concept of memristor, definition of the fractional calculus. The fractional order delayed memristor-based chaotic circuit system is investigated. The Section 3 focus on analysis of stability and Hopf bifurcation analysis in the memristor-based circuit system. The Section 4 presents the Hopf bifurcation and chaos dynamic in the fractional order delayed memristor-based chaotic circuit. The Section 5 concludes the paper.

2. Background

2.1. Memristors

A memristor is a circuital element to show the relationship in the charge (z) and the flux (ϕ). There are two forms of the memristor.

Firstly, the charge-controlled memristor is defined by

$$\begin{cases} v_M = M(z)\, i_M, \\ M(z) = \dfrac{d\phi(z)}{dz}, \end{cases} \tag{1}$$

where $M(z)$ is the memristance.