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Computational methods of phase shifting to stress measurement with photoelasticity using plane polariscope

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A B S T R A C T

The photoelastic technique has seen some renewed interest in past few years with digital images and image processing new methods becoming readily available. However, further research is needed to improve the precision, the accuracy and the automation of photoelastic technique. The aim of this research work is to get new numerical equations for the phase-shifting method in digital photoelasticity using a plane polariscope. The model was developed to plane polariscope because of the simplicity and low cost of this equipment. To develop the phase shift and respective intensity equations only the analyzer is rotated. A ring under diametral compression is used for the experimental validation. From these intensity equations, the equations for isoclinic and isochromatic parameters are deduced by applying a new numerical technique. This approach can be used to calculate the isoclinic and isochromatic parameters using any number of images. Several analysis are performed with different number of photographic images. The results showed errors reduce when more phase-stepped images are utilized. Hence, one concludes that the uncertainties in results due to effects of errors on photoelastic images can be reduced with a larger amount of phase-stepped images.

1. Introduction

The photoelastic technique can be used for whole-field stress analysis in the materials that presents the property of temporary birefringence. The photoelasticity allows determination of the difference between the two principal stresses and the direction of the principal stresses, which are represented by the isochromatic and the isoclinic fringes. The phase-shifting method is one of the most widely used for whole-field analysis. In phase-shifting method, the intensity data for three or more different optical arrangements are recorded. Hence, the fractional fringe order at any point can be determined by the intensity data recorded at that point for various optical arrangements.

In phase-shifting algorithms, the change in phase is achieved by rotation of the optical elements of the polariscope. This requires that all optical elements in polariscope be able to rotate independently. Unfortunately, this is not the case for many commercially available polariscopes widely used in laboratories of industries and universities. The significant advantage of the methodology proposed in this paper is that the method only changes the angle of the analyzer in the polariscope

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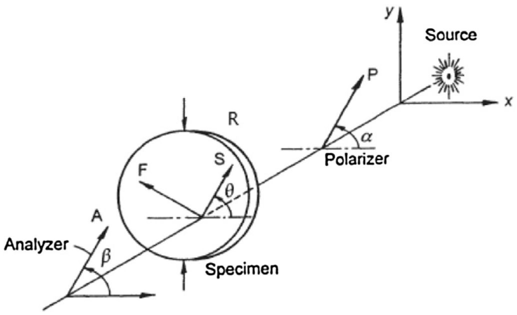

Fig. 1. Optical arrangement of a plane polariscope [14].

and that one can obtain equations for estimating the phase for any number of images in various situations. The physical reason for the proposed numerical model is that the measurement uncertainty can be reduced by increasing the number of phase-stepped images.

Measurement uncertainty is a parameter characterizing the dispersion of the values attributed to a measured quantity. No measurement is exact, the uncertainty has probabilistic basis and reflects incomplete knowledge of the quantity. All the measurements are subject to uncertainty, and a measured value is only complete if a statement of the associated uncertainty accompanies it [8]. The new method can be used with any number of photoelastic measures in plane polariscope.

The main advantages of the plane polariscopes are immune to mismatch of quarter-wave plate, need fewer adjusts during experiments and reduce the chance of error because they have fewer optical elements than circular polariscopes. Furthermore, it was shown that plane polariscope based algorithms give better isoclinics values than the methods that use a quarter wave plate [13,17]. With better results in isoclinic values, it is possible to obtain less error in separation of stresses by the shear stress technique. For this reason, it is important to improve the technique elaborated in [8] initially developed for circular polariscope arrangement to extend it for the plane polariscope arrangement. Another possibility would be to use the two techniques for developing an analogous technique to ten-step method, which is the most adequate for use in manual polariscope based phase-shifting technique [12].

In this paper, a new method to overcome the limitation of positioning of the commercial polariscopes involving a new numerical model is suggested. A comparative study was made between the theoretical results and the numerical methodologies using a ring under diametral compression.

2. The phase-shifting methods

The plane polariscope is one of the simplest optical arrangements possible in photoelastic technique. The Fig. 1 shows the typical arrangement [14], where P, R and A represent polarizer, retarder (stressed model) and analyzer, respectively. The subscripts written after P and A means the angle between the polarizing axis and the reference axis x, for instance: $P_\alpha R_{\theta,\delta} A_\beta$ indicates that the polarizer is at α and analyzer is at an arbitrary angle β with x-axis. The subscripts δ and θ of R indicate the retardation added by stressed sample and whose fast axis is at angle θ with x-axis.

The optical elements of the polariscopes introduce rotation and retardation on the light waves. Using the Jones calculus any arrangement of the plane polariscope is obtained using the matrices operation. The output electric field for the arrangement $P_\alpha R_{\theta,\delta} A_\beta$, as shown in Fig. 1, both along and perpendicular to the analyzer axis (E_β, $E_{\beta+90^\circ}$) are given as [14]

$$\begin{pmatrix} E_\beta \\ E_{\beta+90^\circ} \end{pmatrix} = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\frac{\delta}{2} - i\sin\frac{\delta}{2}\cos 2\theta & -i\sin\frac{\delta}{2}\sin 2\theta \\ -i\sin\frac{\delta}{2}\sin 2\theta & \cos\frac{\delta}{2} + i\sin\frac{\delta}{2}\cos 2\theta \end{bmatrix} \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} K e^{i\omega t}, \tag{1}$$

where $i = \sqrt{-1}$. The symbols K and ω are the amplitude and the angular frequency of the light vector, respectively. To find the output light intensity the following operation is made [14]:

$$I = E_\beta E_\beta^*. \tag{2}$$

In Eq. (2), I is the output light intensity and E_β^* is the complex conjugate of E_β. After operation using Eq. (2), the output intensity emerging from analyzer of the polariscope arrangement $P_\alpha R_{\theta,\delta} A_\beta$ is given by [14]

$$I = I_o \left[\left(\cos\frac{\delta}{2} \right)^2 \left(\cos\left(\beta - \alpha\right) \right)^2 + \left(\sin\frac{\delta}{2} \right)^2 \left(\cos\left(\beta + \alpha - 2\theta\right) \right)^2 \right], \tag{3}$$

where I_o accounts for the amplitude of incident light vector and the proportionality constant. The effect of background intensity is not explicitly taken into account by introducing a parameter I_b in the intensity equation. Instead, we capture an