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# Effect of optical system and turbulent atmosphere on the average intensity of partially coherent flat-topped vortex hollow beam

Dajun Liu*, Xixian Luo, Hongming Yin, Guiqiu Wang, Yaochuan Wang*

*Department of Physics, Dalian Maritime University, Dalian, 116026, China*

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ABSTRACT

Based on the extended Huygens-Fresnel diffraction integral formula, the average intensity of partially coherent flat-topped vortex hollow beam propagating through the paraxial ABCD optical system in atmosphere turbulence are investigated by using the numerical examples. The influences of optical system parameters, coherence length, parameters of turbulent atmosphere, beam order N and topological charge M of laser beam on the average intensity of partially coherent flat-topped vortex hollow beam are analyzed. Results show that the beam will evolve into Gauss-like beam in the far field with the propagation distance increasing, and the beam will evolve into Gauss-like beam more rapidly with the increasing of the structure constant of turbulent atmosphere or the decreasing of coherence length. The results of this work have the potential applications in free space optical communication.



## 1. Introduction

In the recent years, the propagation properties of various laser beams in atmosphere turbulence have been widely studied due to their essential applications in free-space optical communication and remote sensing [1]. And the optical system used in experiment will influence the evolution properties of laser beams propagating in turbulent atmosphere. Since the propagation of a laser beam propagation through an optical system in turbulent atmosphere is studied [2], various laser beam propagation through an optical system in turbulent atmosphere have been investigated, such as cosh-Gaussian beam [3], random electromagnetic beam [4], partially coherent cosine-Gaussian [5], stochastic electromagnetic Gaussian Schell-model beam [6], radially polarized partially coherent beam [7], four-petal Gaussian [8], cylindrical vector Laguerre-Gaussian beam [9], elliptical Gaussian beam [10], annular vortex beam [11], four-petal Gaussian vortex beam [12] et al. and with the development of laser technology, a new beam called flat-topped vortex hollow beam has been introduced and studied widely [13–15]. However, owing to its interesting properties and potential applications, the vortex beam has been widely studied. And the propagation properties of vortex beam through an optical system in turbulent application have the significant meaning for its applications. Then in this paper, we have derived the analytical expressions of partially coherent flat-topped vortex hollow beam propagating through an optical system with two thin lenses in atmosphere turbulence, and studied the influence of optical system, structure constant of turbulent atmosphere, coherence length and the beam parameters N

* Corresponding authors.
*E-mail addresses:* liudajun@dlmu.edu.cn (D. Liu), ycwang@dlmu.edu.cn (Y. Wang).

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and M on the average intensity of partially coherent flat-topped vortex beam propagating through a double-lens system in atmosphere turbulence.

## 2. Propagation theory

Based on the extended Huygens-Fresnel diffraction integral formula, the propagation eqation of laser beam through the ABCD optical system in atmosphere turbulence can be written as [2–12]:

$$E(\mathbf{r},L)=\frac{1}{i\lambda B}\exp(ikL)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}E(\mathbf{r}_0,0)\times\exp\left\{-\frac{ik}{2B}\left[A\left(x_0^2+y_0^2\right)+D\left(x^2+y^2\right)-2\left(xx_0+yy_0\right)\right]+\psi(\mathbf{r}_0,\mathbf{r})\right\}dx_0dy_0 \tag{1}$$

where $\lambda$ is the wavelength; A, B and D are the geometrical ray-matrix elements for the optical system, respectively; $\psi(\mathbf{r}_0,\mathbf{r})$ is the solution to the Rytov method that represents the random part of the complex phase; $\mathbf{r}=(x,y)$ and $\mathbf{r}_0=(x_0,y_0)$ are the position vectors at the output and the input planes, respectively. D = 1 represents that the dimension of the lens are assumed to be larger than the beam diameter.

Based on the theory of coherence, the average intensity of partially coherent laser beams propagating through the ABCD optical system in atmosphere turbulence can be expressed as:

$$\langle I(\mathbf{r},z)\rangle=\frac{1}{\lambda^2B^2}\iiiint_{-\infty}^{+\infty}E_x(\mathbf{r}_{10},0)E_x^*(\mathbf{r}_{20},0)\times\exp\left\{-\frac{ik}{2B}\left[A\left(x_{10}^2-x_{20}^2+y_{10}^2-y_{20}^2\right)-2\left(xx_{10}-xx_{20}+yy_{10}-yy_{20}\right)\right]\right\}\times\langle\exp\left[\psi(\mathbf{r}_{10},\mathbf{r})+\psi^*(\mathbf{r}_{20},\mathbf{r})\right]\rangle d\mathbf{r}_{10}d\mathbf{r}_{20} \tag{2}$$

where the asterisk $*$ denotes the complex conjugation. And

$$\langle\exp\left[\psi(\mathbf{r}_{10},\mathbf{r})+\psi^*(\mathbf{r}_{20},\mathbf{r})\right]\rangle=\exp\left[-\frac{(x_{01}-x_{02})^2+(y_{01}-y_{02})^2}{\rho_0^2}\right] \tag{3}$$

with $\rho_0$ is the spherical-wave lateral coherence radius due to the turbulence of the entire optical system and is defined as [2]

$$\rho_0=B\left[1.46k^2C_n^2\int_0^L b^{5/3}(z)\,dz\right]^{-3/5} \tag{4}$$

where $C_n^2$ is the constant of refraction index structure of atmosphere turbulence and which describes the turbulence strength of atmosphere turbulence. $\rho_0$ is the coherence length (induced by the atmosphere turbulence) of a spherical wave propagating in the atmosphere turbulence. $b(z)$ corresponds to the approximate matrix element for a ray propagating backwards through the system. z is the axial distance between the source plane and the output plane.

Based on the theory of coherence, the electric field of partially coherent flat-topped vortex hollow beam at the source plane z = 0 can be written as [14]:

$$W(\mathbf{r}_{10},\mathbf{r}_{20},0)=\sum_{m=1}^{N}\sum_{n=1}^{N}\frac{(-1)^{m+n}}{N^2}\binom{N}{m}\binom{N}{n}\exp\left[-n\left(\frac{x_{10}^2}{w_x^2}+\frac{y_{10}^2}{w_y^2}\right)\right]\exp\left[-m\left(\frac{x_{20}^2}{w_x^2}+\frac{y_{20}^2}{w_y^2}\right)\right]\times\left(\frac{x_{10}x_{20}}{w_x^2}+\frac{y_{10}y_{20}}{w_y^2}+i\frac{x_{20}y_{10}-x_{10}y_{20}}{w_xw_y}\right)^M\exp\left[-\frac{(x_{10}-x_{20})^2+(y_{10}-y_{20})^2}{2\sigma^2}\right] \tag{5}$$

where $N$ is the order of partially coherent flat-topped vortex hollow beam, $M$ is the topological charge; $w_x$ and $w_y$ are the beam waist width in x and y directions, respectively; $\binom{N}{n}$ denotes the binomial coefficient; with $\mathbf{r}_{10}=(x_{10},y_{10})$ and $\mathbf{r}_{20}=(x_{20},y_{20})$ represent the position vectors at the source plane.

By substituting partially coherent flat-topped vortex hollow beam equation (5) into equation (2), and recalling the following formula [16]

$$\int_{-\infty}^{+\infty}x^n\exp\left(-px^2+2qx\right)dx=n!\exp\left(\frac{q^2}{p}\right)\left(\frac{q}{p}\right)^n\sqrt{\frac{\pi}{p}}\sum_{k=0}^{[\frac{n}{2}]}\frac{1}{k!(n-2k)!}\left(\frac{p}{4q^2}\right)^k \tag{6}$$