Optik xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Evanescent spherical waves of scalar point source

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ARTICLE INFO

Article history:
Received 16 August 2016
Accepted 19 October 2016

Keywords:
Spherical wave
Evanescent wave
Homogeneous wave
Weyl's expansion

ABSTRACT

It is well known that evanescent waves decay exponentially from a wave source and influence on a near field of the particular source. In a zero frequency limit these waves decay according to a power law around a spherical source. In the paper exact calculations of an evanescent and homogeneous field in the zero frequency limit has been performed. The calculations base on an angular spectrum representation of a spherical wave using Weyl's expansion technique.

1. Introduction

Evanescent waves are of great interest in a modern science [1]. In contemporary measurements and signal analysis, near field detecting techniques exploits the benefit of physical properties of the evanescent wave. In this view, it is essential to know the properties of the evanescent wave in order to properly detect and evaluate the received signals which are further used in various Fourier wave techniques [2,3].

Exponential decay radial spherical evanescent waves appear outside an oscillating sphere. In the zero frequency limit this waves undergo power law decay. Initial research of this phenomenon has been triggered by Toraldo di Francia [4] and latter by Asby and Wolf [5] who studied a charged particle uniformly moving in vacuum. By approaching a charged particle velocity to zero, the zero frequency limit is studied by Agudin and Platzeck [6,7]. In present paper the exact calculation of the evanescent and homogeneous field in the zero frequency limit has been performed on the base of Weyl's expansion technique [8].

2. Spherical outgoing waves

The calculation of spherical waves can be achieved by solving the Helmholtz differential equation [2]

$$\frac{1}{r^2\Psi}\frac{\partial}{\partial r}(r^2\frac{\partial\Psi}{\partial r})+\frac{1}{r^2\Psi\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\Psi}{\partial\theta})+\frac{1}{r^2\Psi\sin^2\theta}\frac{\partial^2\Psi}{\partial\phi^2}=-k^2, \tag{1}$$

where r, θ and ϕ are spherical coordinates, Ψ is scalar function and $k=\omega/c$ is a wave number for the given wave velocity c and angular frequency ω. The outgoing spherical waves of a scalar point source can be well described as the solution of the Helmholtz equation with a zero order spherical Hankel function of the first kind $h_0^{(1)}$

$$h_0^{(1)}(kr)=j_0(kr)+iy_0(kr). \tag{2}$$

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http://dx.doi.org/10.1016/j.ijleo.2016.10.065

This equation contains a zero order spherical Bessel function of the first kind $j_0(kr)$ and zero order spherical Bessel function of the second kind $y_0(kr)$ respectively [9]. Explicitly written, the zero order spherical Bessel functions are

$$j_0(kr) = \frac{\sin(kr)}{kr}, \tag{3}$$

$$y_0(kr) = -\frac{\cos(kr)}{kr}. \tag{4}$$

In order to represent the spherical waves as plane waves, an angular spectrum representation is used by using Weyl's integral formula as follows.

3. Angular spectrum representation

According to Eqs. (2), (3) and (4), the Hankel function can be written in exponential form

$$h_0^{(1)}(kr) = \frac{\exp(ikr)}{ikr}. \tag{5}$$

Furthermore $h_0^{(1)}(kr)$ can be expressed in terms of α and β angle [10]

$$\frac{\exp(ikr)}{ikr} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} \exp\left[ik(x\sin\alpha\cos\beta + y\sin\alpha\sin\beta \pm z\cos\alpha\right]\sin\alpha\, d\alpha\, d\beta\,, \tag{6}$$

where upper + sign is connected with a positive z coordinate and lower – sign with negative z coordinate. Here α denotes the angle between z coordinate and plane wave direction and β denotes the angle between x coordinate and plane wave direction. In order to simplify the equation, new variables k_x, k_y and k_z can be introduced

$$\begin{cases} k_x = k\sin\alpha\cos\beta, \\ k_y = k\sin\alpha\ \sin\beta, \\ k_z = k\cos\alpha, \end{cases} \tag{7}$$

where $k_z = \sqrt{k^2 - {k_x}^2 - {k_y}^2}$ and $r^2 = x^2 + y^2 + z^2$. By inserting (7) into the integral Eq. (6) one can achieve simpler form

$$\frac{\exp(ikr)}{ikr} = \frac{1}{2\pi k}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tfrac{1}{k_z}\exp\left[i(k_x x + k_y y + k_z|z|)\right] dk_x\, dk_y\,. \tag{8}$$

The complete wave field $G = \exp(ikr)/r$ can be expressed as a sum of the homogeneous or propagating spherical field when $k_z = \sqrt{k^2 - {k_x}^2 - {k_y}^2}$ if $k_x^2 + k_y^2 \le k^2$ and evanescent or decaying spherical field when $k_z = i\sqrt{{k_x}^2 + {k_y}^2 - k^2}$ if $k_x^2 + k_y^2 \ge k^2$

$$G = G_h + G_e. \tag{9}$$

The calculation of the evanescent and homogeneous scalar spherical field can be achieved from integral (8) [11]

$$G_e(x,y,z) = \frac{i}{2\pi}\iint_{k_x^2+k_y^2\ge k^2} \frac{1}{k_z}\exp[i(k_x x + k_y y + k_z|z|)]dk_x\, dk_y\,,\ \ k_z = i\sqrt{{k_x}^2 + {k_y}^2 - k^2}, \tag{10}$$

$$G_h(x,y,z) = \frac{i}{2\pi}\iint_{k_x^2+k_y^2\le k^2} \frac{1}{k_z}\exp[i(k_x x + k_y y + k_z|z|)]dk_x\, dk_y\,,\ \ k_z = \sqrt{k^2 - {k_x}^2 - {k_y}^2}. \tag{11}$$

The solution of the integral (10) is according to [12–14]

$$G_e(x,y,z) = \frac{1}{r}[2U_0\left(kr - k|z|, k\sqrt{x^2+y^2}\right) - J_0\left(k\sqrt{x^2+y^2}\right)]. \tag{12}$$

The second integral (11) can be calculated from identity (9)

$$G_h(x,y,z) = \frac{1}{r}\left[\exp(ikr) + J_0\left(k\sqrt{x^2+y^2}\right) - 2U_0\left(kr - k|z|, k\sqrt{x^2+y^2}\right)\right]. \tag{13}$$

Here J_0 is the zero order Bessel function

$$J_0(k\sqrt{x^2+y^2}) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\exp(-ik\sqrt{x^2+y^2}\sin\tau)d\tau, \tag{14}$$