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Propagation of matter wave in a medium

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ABSTRACT

Propagation of matter wave in a medium is found to be governed by a damped Klein–Gordon (or telegraph) equation. This equation and Dirac equation are treated as describing a wavepacket propagating at speed of light. Matter fields analogous to electric and magnetic fields that are associated with the electric charge are found to be associated with particle mass. In this formulation the matter magnetic field vanishes. Moreover, the mass of the moving damped spin – 0 particle in a conducting medium is found to be related to the conductivity of the medium. The electromagnetic field of massive photon following Klein–Gordon equation in a conducting medium is created from the spatial and temporal variations of the charge and current densities. The violation of Lorenz gauge condition is found to give rise to longitudinal wave with zero spin.

1. Introduction

Relying on de Broglie hypothesis [1], Schrodinger formulated his wave quantum theory by assigning a wavefunction for the development (state) of quantum particle, like an electron [2]. Owing to Heisenberg uncertainty principle the quantum particle cannot be determined exactly. According to Born theory, the square of this wavefunction defines the probability of finding the particle at a given position at a particular time. On the other hand, a single wave cannot describe a particle as a plane wave extend over an infinitely large region is space. Hence, a particle should be described by a number of individual waves forming a wavepacket.

While a phase velocity is referred to the velocity of a particular point in the wavefront, the group velocity expresses the overall velocity of the wavepacket. In general, these two velocities are different. The equivalence of mass and energy of Einstein supports the de Broglie wave – particle duality. Particle and wave nature can't be exhibited at the same time of a quantum particle, but they are complement to each other. A moving particle is thus exhibiting a perpetual process of creation and annihilation. The time interval between creation and annihilation can be determined from quantum mechanics laws. Therefore, a moving particle will appear to be jolting. This is what Schrodinger had called *zitterbewegung*, when he applied the wave packet solutions for an electron moving in free space using Dirac equation [4,3]. He attributed this effect to interference between positive and negative energy states of the moving electron.

In Maxwell theory the photon is massless when propagating in free space. However, the photon could be massive in propagating in a medium. This is because photons interact with electron inside a medium and thus acquire a mass. This case shows up when light get refracted by a medium where its velocity becomes different from that in free space. As a result the photon should be described by a quantum theory as well as Maxwell theory. These also support the concept of photon duality.

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In this work we study the motion of photons in vacuum and in a medium. The electrical signals are known to travel in electric circuit according to a telegraph equation [5]. Hence, energy flows both inside the circuit (wire), and in the space around it. The energy inside the wire experiences a dissipation since its carried by massive photons. Massive photons are described by Proca equations that generalize Maxwell equations [6]. This motion is exhibited in superconductivity where the electromagnetic interaction becomes of short range.

The first theory of superconductivity was advocated by London [7]. However, London theory was classical since he used Newton's law and Maxwell equations. It was successful in describing Meissner effect of magnetic field [8]. In the present study we provide the quantum London theory. It reflects the propagation of a spin – 0 massive photon that is governed by Maxwell and quantum theories. We show that superconductivity is transmitted by spin zero massive photons. It is believed that the carriers of superconductivity are Cooper pairs. We thus trust that Cooper pairs are the massive photons.

2. Dirac and Klein–Gordon equations

The spin – 1/2 particle is governed by Dirac equation [2,4]

$$\frac{1}{c}\frac{\partial\psi}{\partial t}+\vec{\alpha}\cdot\vec{\nabla}\psi+\frac{imc\beta}{\hbar}\,\psi=0\,, \tag{1}$$

while spin – 0 particle is govern by the Klein–Gordon equation [2]

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}-\nabla^2\phi+\frac{m^2c^2}{\hbar^2}\,\phi=0\,, \tag{2}$$

where $\vec{\alpha}$ and β are Dirac matrices, m and p are the rest mass and relativistic momentum of the particle, respectively. The wavefunction ψ is neither a vector nor a scalar, it is called a spinor and has 4 – components. The wavefunction ϕ is a scalar and has only one component. Eqs. (1) and (2) admit a plane wave solution. These equations are statement of conservation of relativistic energy. The total relativistic energy in Dirac is given by

$$E=c\,\vec{\alpha}\cdot\vec{p}+\beta\,mc^2\,, \tag{3}$$

while in Klein–Gordon is given by

$$E=\sqrt{c^2p^2+m^2c^4}\,, \tag{4}$$

as can be seen from Eqs.(1) and (2). Since a single wave cannot be describe a particle, the group velocity of the particle (wavepacket) is defined by [9,10]

$$v_g=\frac{\partial E}{\partial p} \tag{5}$$

while the phase velocity is defined by

$$v_{ph}=\frac{E}{p}\,. \tag{6}$$

The group velocity of a Klein–Gordon particle is thus

$$v_g=\frac{p\,c^2}{E}\,, \tag{7}$$

while for Dirac particle it is given by

$$v_g=\alpha\,c\,,\qquad v_g=\pm\,c\,, \tag{8}$$

since the eigen values of α are ± 1. This implies that the particle is described by a wavepacket consisting of two waves traveling to the left and right with speed of light.

3. Unified quantum mechanics

A quaternionic formation of quantum mechanics yields a system of equations expressed as [11,12]

$$\vec{\nabla}\cdot\vec{\psi}-\frac{1}{c^2}\frac{\partial\psi_0}{\partial t}-\frac{m}{\hbar}\,\psi_0=0\,, \tag{9}$$

$$\vec{\nabla}\psi_0-\frac{\partial\vec{\psi}}{\partial t}-\frac{mc^2}{\hbar}\,\vec{\psi}=0\,, \tag{10}$$

and

$$\vec{\nabla}\times\vec{\psi}=0\,. \tag{11}$$