Optik 137 (2017) 167–174

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Observer-based control on a chaotic system with unknowns and uncertainties

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ARTICLE INFO

Article history:
Received 3 December 2016
Accepted 21 February 2017

Keywords:
Sliding mode control
Uncertainties
Unknowns
Lyapunov method
Observer

ABSTRACT

This paper investigate the control problem for a kind of chaotic system in the case that uncertainties and unknowns exist; one observer is constructed to realize the estimate of the system state variable, unknowns and uncertainties effectively; then based on sliding mode technique and Lyapunov stability theory, a controller is designed to achieve the trajectory tracking control; some typical numerical simulation examples are included at last to illustrate the effectiveness of the given theoretical results.

1. Introduction

Control on nonlinear system has been a hot research area. Up to now, many methods have been presented, such as differential geometry method [1,2], fuzzy control method [3–6], sliding mode method [7–10] and so on. Generally these methods are on the basis of multidimensional control input, which not only will increase the implementation costs, but also may introduce new or more disturbances. Hence, how to construct the single-dimensional controller to realize the control for the nonlinear system has become a challenging issue.

As a special nonlinear system, chaotic system has received many attentions from scholars for its good characteristics and potential application. For instance, in [11], an improved unidirectional coupling correlation control method is proposed to realize the control of the undesirable chaotic behavior in Permanent magnet synchronous motor; in [12], the problem on the global asymptotical synchronization problem of chaotic Lur′e systems is addressed by using a delayed feedback proportional-derivative control scheme; in [13], a prediction-based control method is presented to achieve the synchronization of chaotic energy resource systems; in [14], adaptive approach is proposed to realize the synchronization for a hyper-chaotic system.

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http://dx.doi.org/10.1016/j.ijleo.2017.02.079

However, these methods just focus on one kind of chaotic system, which limits their applied scope. In this paper, the following chaotic system is considered

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \quad \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f(x) \end{cases} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in \mathbf{R}^n$, $f(x)$ is nonlinear term. Above chaotic systems denote a class of chaotic systems. Many chaotic systems, such as Lorenz system, Chen's system, Lu's system and so on, can been transformed into above system through topology transformation. Compared with special chaotic systems, the investigation on chaotic system (1) will have more application values.

In addition, uncertainties and unknowns exist in real systems widely, and usually, only one measured system output is available, which make the real control problem more complicated. Observer is a powerful tool to solve this problem [15–17], however the researches on the control of chaotic systems with uncertainties and unknowns based on one observer, which is used to realize the estimate of system state variable, uncertainties and unknowns, are seldom carried out. All above these motivate our research.

In this paper, we will focus on the control problem for chaotic system (1) with uncertainties and unknowns based on one observer. Model description and preliminaries will be given in Section 2. Main theoretical results will be derived in Section 3. The typical numerical example will be included to verify the correctness of our theoretical results in Section 4. At last, some conclusions will be presented in Section 5.

Notation: R^n is the n-dimensional Euclidean space. $(\cdot)^{(i)}$ denotes the (i)th derivative of $(\cdot)$. $\|\cdot\|$ refers to the Euclidean vector norm.

2. Model description and preliminaries

In this paper, the following class of systems with uncertainties and unknowns are considered

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \quad \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = l(x) + \Delta h(x) + h(x) + g(x)u \\ y = x_1 \end{cases} \tag{2}$$

where $x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in \mathbf{R}^n$, $u \in \mathbf{R}$ is control input, $y \in \mathbf{R}$ is system output, $l(x)$ is unknown nonlinear term, $\Delta h(x)$ is uncertainties, $h(x)$ is known term, $g(x)$ is system parameter, satisfies $0 < g_m \le g(x) \le g_M$, where g_m and g_M are positive scalars.

Define the tracking error

$$e(t) = x(t) - y_d(t)$$

where $e = (e_1(t), e_2(t), \cdots e_n(t))^T \in R^n$, $y_d(t) = (y_{d1}(t), y_{d2}(t), \cdots, y_{dn}(t))^T \in R^n$, and $y_{d1}(t)$ is the desired trajectory with

$$y_{di}(t) = {y_{d1}}^{(i-1)}(t), \; i \le n$$

where $(\cdot)^{(i-1)}$ denotes the $(i-1)$th derivative of $(\cdot)$.

Then the tracking error dynamic system can be expressed by

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \quad \vdots \\ \dot{e}_{n-1} = e_n \\ \dot{e}_n = l(x) + \Delta h(x) + h(x) + g(x)u - {x_r}^{(n)} \end{cases} \tag{3}$$

where $e_i = x_i - {y_d}^{(i-1)}$, $i = 1, 2, \cdots, n$.