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Maxwellian quantum mechanics

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ABSTRACT

Expanding the ordinary Dirac's equation in quaternionic form yields Maxwell-like field equations. As in the Maxwell's formulation, the particle fields are represented by a scalar, ψ_0 and a vector $\vec{\psi}$. The analogy with Maxwell's equations requires that the inertial fields are $\vec{E}_D = c^2\vec{\alpha} \times \vec{\psi}$, and $\vec{B}_D = \vec{\alpha}\psi_0 + c\beta\vec{\psi}$ and that $\psi_0 = -c\beta\vec{\alpha} \cdot \vec{\psi}$, where β, $\vec{\alpha}$ and c are the Dirac matrices and the speed of light, respectively. An alternative solution suggests that magnetic monopole-like behavior accompanies Dirac's field. In this formulation, a field-like representation of Dirac's particle is derived. It is shown that when the vector field of the particle, $\vec{\psi}$, is normal to the vector $\vec{\alpha}$, Dirac's field represents a medium with maximal conductivity. The energy flux (Poynting vector) of the Dirac's fields is found to flow in opposite direction to the particle's motion. A system of equivalently symmetrized Maxwell's equations is introduced. A longitudinal (scalar) wave traveling at speed of light is found to accompany magnetic charges flow. This wave is not affected by presence of electric charges and currents. The Lorentz boost transformations of the matter fields are equivalent to $c\vec{\psi}' = c\vec{\psi} \pm \beta\vec{\alpha}\psi_0$, $\psi_0' = \psi_0 \mp c\beta\vec{\alpha} \cdot \vec{\psi}$.

1. Introduction

Maxwell had unified the laws of electricity and magnetism in a consistent way into a set of four equations [1]. The sources of the electric and magnetic fields are the charges and currents (moving charges). De Broglie had postulated that a moving particle exhibits a wave-like nature. This wave is concomitant with the particle as it moves on in a form of a wave packet centered around the particle. On the other hand, the electric and magnetic fields associated with the moving charges and currents spread away in space and have oscillatory behavior. Biot–Savart showed that the electric and magnetic fields due to a charged particle moving at constant speed are stationary and centered around the particle. In quantum mechanics, Dirac and Schrodinger equations govern the motion of a particle (e.g., an electron) owing to its mass (matter) nature irrespective of its charge. However, a moving charged particle can interact with the electromagnetic field existing in space. The proper formation is described by electrodynamics merging quantum mechanics with Maxwell's theory.

The question that normally arises is that how do the two waves interact, the matter waves (due to mass) and the field waves (due to charge). Moreover, one can also think of another wave nature, that is governed by some equation, pertaining to another physical property of the particle, like spin, instead of incorporating it in the former formulations.

In a recent paper, we have derived the ordinary Dirac equation (in an unfamiliar form), Klein-Gordon, and Schrodinger equations from a quaternionic form of Dirac's equation [2]. Earlier, we have made an analogy between hydrodynamics and electrodynamics and showed that hydrodynamics laws can be written in Maxwell-like fields equations. Following a same line of reasoning, we have presented an analogy between matter (de Broglie) waves and the electromagnetic waves

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[3]. Moreover, a quaternionic Maxwell's equation is found to allow for scalar (longitudinal) waves besides the ordinary electromagnetic transverse waves [4]. The use of quaternions in formulating physical laws is found to be very rich [5,6].

In this work we extend our formulation to investigate another form of quaternionic Dirac's equation. We aim in this work to describe the Dirac particle (matter nature) by assigning a field character rather than a matter-wave character. We then compare the equations governing these fields with the corresponding Maxwell's equations. This approach will enhance deepening the notion of the duality nature exhibited by micro-particles. However, a rather complete wave-particle duality is recently investigated [7]. The new quaternionic Dirac's equation formulation is found to be very promising in bringing the above ideas into reality. The resulting set of equations emerging from the quaternionic Dirac's equation is analogous to Maxwell's electromagnetic field equations.

We introduce the classical electromagnetism in Section 2, and the quaternionic Dirac's equation in Section 3. We then define the analogous electric and magnetic matter fields, and compare the latter field with that of the Biot–Savart fields arising from a charge moving at constant velocity. In Section 4 we provide the energy and momentum densities of the two analogous fields, and showed that they are described by the same set of equations. We have found that while the electromagnetic fields can dissipate energy in some cases, the matter fields spread without energy loss thus preserving the particle integrity. The direction of the matter field flux density is opposite to the particle's motion. A symmetrized Dirac's field equations are obtained by relaxing one of our conditions on the Dirac's fields. This latter condition allows for longitudinal wave nature for the Dirac's field and permits magnetic monopoles to exist. This longitudinal (scalar) wave, Λ, has an energy flux $-\Lambda\vec{B}$ and energy density $(\varepsilon_0/2)\Lambda^2$.

2. Maxwell's fields equations

The force on a charged particle q in the presence of electric $\vec{E}$, and magnetic field $\vec{B}$, is given by Lorentz force as

$$\vec{F}_L = q(\vec{E} + \vec{v} \times \vec{B}). \tag{1}$$

According to the Newton's second law of motion the particle (charge) acceleration $\vec{a}$ is given by $\vec{F}_L = m\vec{a}$. However, the electromagnetic fields are determined by Maxwell's equations [1],

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{2}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}, \tag{3}$$

and

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}, \tag{4}$$

$$\vec{\nabla} \cdot \vec{B} = 0. \tag{5}$$

Here $\vec{J}$ and ρ represent the source of the electromagnetic fields. In Maxwell's formulation the matter field has not been considered, as this was not yet known at that time.

We would like now to employ quaternions to rewrite Dirac's equation, and to compare the resulting equations with Maxwell's equations. This is because the electron charge nature is expressed by Maxwell's equations, while its wave nature is described by Dirac. To this aim, we should express Dirac particles by matter fields rather than wavefunctions (spinors).

3. The quaternionic Dirac's equation

The ordinary Dirac's equation of a spin-1/2 particle with rest mass m is expressed as [8]

$$p^\mu \gamma_\mu \psi = mc\psi, \tag{6}$$

where γ^μ are expressed in terms of Pauli matrices, c is the speed of light, and ψ are the spinors representing the Dirac's wavefunction. In quaternionic form Eq. (6) reduces to

$$\tilde{P}\tilde{\gamma}\tilde{\Psi} = mc\tilde{\Psi}, \tag{7}$$

where

$$\tilde{P} = \left(\frac{i}{c}E, \vec{p}\right), \quad \tilde{\gamma} = (i\beta, \vec{\alpha}), \quad \tilde{\Psi} = \left(\frac{i}{c}\psi_0, \vec{\psi}\right). \tag{8}$$

In quantum mechanics the above energy and momentum become operators, viz., $\vec{p} = -i\hbar\vec{\nabla}$ and $E = i\hbar(\partial/\partial t)$. Here ψ_0 and $\vec{\psi}$ are the Dirac's particle fields which we will shortly give them a meaning. To envisage this relation, we know that the Dirac spinor can be decomposed into 2×2 components (e.g., Weyl). From group point of view, one has $2\otimes 2 = 1\oplus 3$. Here $\vec{\psi}$ represents the 3-vector and ψ_0 the 1-scalar. Hence, the 4-components spinor is decomposed in a scalar and a vector. Some authors express Maxwell's equations in a Dirac-like form [9,10]. They employ a complex vector, $\vec{F} = (\vec{E}/c) + i\vec{B}$. This vector