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A compact finite scheme for derivatives calculation of electrostatic field in image tube

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ABSTRACT

In this paper, an eighth order combined compact finite difference scheme was presented. Through Fourier analysis and numerical tests, one can find that the scheme has spectral-like resolution, and proved to be an ideal class of schemes for the calculation of high order derivatives calculation of electrostatic field in image tube.

1. Introduction

The streak camera has been successfully developed as an instrument for the recording of ultrafast transient phenomenon with ultrahigh temporal and spatial resolution in the picoseconds or sub-picoseconds regime, with the performance of streak camera was affected by the aberrations of the image tube, the core components of streak camera. According to Zhou [1], the aberrations calculation of image tube were due to the second and even higher order derivatives of electrostatic which cannot be calculated directly, since there is not analytical solution.

Finite difference scheme is the usual simulation method used to solve the Laplace Equation through the Taylor expansion in two or three dimension, and finally figure out the voltage distribution of image tube by iteration. Once the physical values of voltage distributions were calculated, one need also computed the derivatives (at least from first to fourth for third-level aberration calculations) accurately. The common used second order central scheme was insufficient, means that we need higher order precision schemes.

According to the Taylor expansion, the higher accuracy for finite difference scheme, the more nodes needed. In contract, compact finite difference scheme can achieve the same order or even higher order precision with less nodes compare to normal finite difference schemes. The most influential compact scheme is the Lele's schemes [2], which proved to have spectral-like resolution for short waves by Fourier analysis. Then Mahesh [3] developed a family of compact schemes coupling the second derivatives also proved to be good resolution. Based on Zhang [4] developed a new class of central compact schemes using both the values of cell centers and grid nodes, improved both the accuracy order and the wave resolution excellently. However, the Lele's schemes involve at least five nodes (when $c=0$) or seven nodes (when $c \neq 0$), means need at least two more near boundary schemes; Shu Hai Zhang's schemes use the extrapolation method to derive the boundary schemes, which was complicated for keeping high order globally; Mahesh deduced the boundary schemes by

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Taylor expansion, but only four order precision for the second boundary derivatives matching with the sixth or eighth order interior schemes.

In this paper, an eighth order combined compact finite scheme based on Mahesh's schemes and using the methodology of Zhang [4] was presented. Through systematic Fourier analysis, it was proved to be spectral like resolutions, and the numerical test shows that it was an ideal scheme for the calculation of high order derivatives of electrostatic field in image tube.

This paper is organized as follows. Section 2 presents the method to design our scheme. Section 3 contains a systematic Fourier analysis to analyze the wave resolution of our schemes. Section 4 designs the boundary closures schemes. Section 5 presents the numerical test of our schemes. In Section 6, we apply our schemes to compute the high order derivatives of practical electrostatic image tube. The concluding remarks are made in Section 7.

2. Combined compact schemes

In this section, a compact finite scheme was derived from the combined finite difference schemes proposed by Mahesh [3], then was extended to be an eighth order scheme with spectral like resolution by using both the methodology of Shu Hai Zhang [4] and the cell interpolation schemes of Lele [2].

$$\begin{aligned}&a_1 f_{i-1}^{(N+2)}+a_2 f_{i-1}^{(N+1)}+a_3 f_i^{(N+1)}+a_4 f_i^{(N+2)}+a_5 f_{i+1}^{(N+1)}+a_6 f_{i+1}^{(N+2)}\\&=b_1 f_{i-1}^{(N)}+b_2 f^{(N)}_{i-\frac{1}{2}}+b_3 f_i^{(N)}+b_4 f^{(N)}_{i+\frac{1}{2}}+b_5 f_{i+1}^{(N)}(N=0,2,4\ldots)\end{aligned} \tag{1}$$

Where $f_i^{(N)}$, $f_i^{(N+1)}$, $f_i^{(N+2)}$ represent the (N)th, (N + 1)th, (N + 2)th derivative at node x_i respectively. One can use Eq. (1) to compute the higher order derivatives (N + 1)th and (N + 2)th once the N th derivative was computed.

The scheme given by (1) contains the values on the cell centers, which are unknown and could be obtained using the cell compact interpolation scheme [2].

The coefficients in Eq. (1) were derived by matching the Taylor series coefficient of various orders. Schemes of order ranging from second to eighth could be obtained by solving the resulting set of Taylor expansion equations. In this paper, we represent only the highest order (eighth) schemes for demonstration.

The coefficient of eighth order scheme of the (N + 1) th derivative:

$$\begin{cases}a_{11}=-\dfrac{h}{48},a_{12}=-\dfrac{37}{144},a_{13}=1,a_{14}=0,a_{15}=-\dfrac{37}{144},a_{16}=\dfrac{h}{48}\\ b_{11}=\dfrac{407}{432h},b_{12}=-\dfrac{64}{27h},b_{13}=0,b_{14}=\dfrac{64}{27h},b_{15}=-\dfrac{407}{432h}\end{cases} \tag{2a}$$

The coefficient of eighth order scheme of the (N + 2) th derivative:

$$\begin{cases}a_{21}=\dfrac{1}{24},a_{22}=\dfrac{43}{72h},a_{23}=0,a_{24}=1,a_{25}=-\dfrac{43}{72h},a_{26}=\dfrac{1}{24}\\ b_{21}=-\dfrac{67}{27h^2},b_{22}=\dfrac{256}{27h^2},b_{23}=-\dfrac{14}{h^2},b_{24}=\dfrac{256}{27h^2},b_{25}=-\dfrac{67}{27h^2}\end{cases} \tag{2b}$$

The subscripts a_{1i}, a_{2i}, b_{1i}, b_{2i} of (2a) and (2b) were used to distinguish the coefficients of (N + 1)th and (N + 2)th derivative, with the truncation error $-\frac{h^8 f_i^{(N+9)}}{1451520}$ and $\frac{h^8 f_i^{(N+10)}}{7257600}$ for (N + 1) th and (N + 2) th derivatives respectively.

3. Fourier analysis and error transfer description

Fourier analysis and the 'modified wavenumber' provided a convenient method for quantifying the error. Since ComCS contains the first and second derivatives, the modified wavenumber may figure out to be:

$$\{w^{(1)}=\frac{10752\ \sin(\frac{w}{2})-3372\ \sin(w)-512\ \sin(\frac{3w}{2})+\frac{129}{2}\ \sin(2w)}{1365+2232\ \left(\sin(\frac{w}{2})\right)^2-18\ (\sin(w))^2} \tag{3a}$$

$$\{w^{(2)}=-\frac{259}{6}-\frac{-\frac{1144391}{2}+135168\ \cos(\frac{w}{2})+323748\ \left(\cos(\frac{w}{2})\right)^2-63488\ \left(\cos(\frac{w}{2})\right)^3}{10791-6912\ \left(\cos(\frac{w}{2})\right)^2+216\ \left(\cos(\frac{w}{2})\right)^4} \tag{3b}$$

Once the first and the second derivative were obtained, one could replace the values on the right hand-side stencil to obtain the higher order derivatives, i.e. $w^{(3)}=w^{(1)}*w^{(2)}$, $w^{(4)}=w^{(2)}*w^{(2)}$, $w^{(5)}=w^{(1)}*w^{(4)}$, $w^{(6)}=w^{(2)}*w^{(4)}$,. . . . . .

Fig. 1 shows the modified wavenumber ranged from first to fourth derivative of ComCS and the exact solution.