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Feynman graph generation and calculations in the Hopf algebra of Feynman graphs☆

Michael Borinsky*

Institutes of Physics and Mathematics, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099 Berlin, Germany

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ABSTRACT

Two programs for the computation of perturbative expansions of quantum field theory amplitudes are provided. **feyngen** can be used to generate Feynman graphs for Yang–Mills, QED and φ^k theories. Using dedicated graph theoretic tools **feyngen** can generate graphs of comparatively high loop orders. **feyncop** implements the Hopf algebra of those Feynman graphs which incorporates the renormalization procedure necessary to calculate finite results in perturbation theory of the underlying quantum field theory. **feyngen** is validated by comparison to explicit calculations of zero dimensional quantum field theories and **feyncop** is validated using a combinatorial identity on the Hopf algebra of graphs.

Program summary

Program title: **feyngen**, **feyncop**
Catalogue identifier: AEUB_v1_0
Program summary URL: http://cpc.cs.qub.ac.uk/summaries/AEUB_v1_0.html
Program obtainable from: CPC Program Library, Queen's University, Belfast, N. Ireland
Licensing provisions: Standard CPC licence, http://cpc.cs.qub.ac.uk/licence/licence.html
No. of lines in distributed program, including test data, etc.: 2657
No. of bytes in distributed program, including test data, etc.: 22 606
Distribution format: tar.gz
Programming language: Python.
Computer: PC.
Operating system: Unix, GNU/Linux.
RAM: 64 m bytes
Classification: 4.4.
External routines: nauty [1], geng, multig (part of the nauty package)
Nature of problem:
Performing explicit calculations in quantum field theory Feynman graphs are indispensable. Infinities arising in the perturbative calculations make renormalization necessary. On a combinatorial level renormalization can be encoded using a Hopf algebra [2] whose coproduct incorporates the BPHZ procedure. Upcoming techniques initiated an interest in relatively large loop order Feynman diagrams which are not accessible by traditional tools.
Solution method:
Both programs use the established **nauty** package to ensure high performance graph generation at high loop orders. **feyngen** is capable of generating φ^k-theory, QED and Yang–Mills Feynman graphs and of filtering these graphs for the properties of connectedness, one-particle-irreducibleness, 2-vertex-connectivity and tadpole-freeness. It can handle graphs with fixed external legs as well as those without fixed external legs.
feyncop uses basic graph theoretical algorithms to compute the coproduct of graphs encoding their Hopf algebra structure.

☆ This paper and its associated computer program are available via the Computer Physics Communication homepage on ScienceDirect (http://www.sciencedirect.com/science/journal/00104655).

* Tel.: +49 17647040034.
E-mail address: borinsky@physik.hu-berlin.de.

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Running time:
All 130 516 1PI, φ^4, 8-loop diagrams with four external legs can be generated, together with their symmetry factor, by **feyngen** within eight hours and all 342 430 1PI, QED, vertex residue type, 6-loop diagrams can be generated in three days both on a standard end-user PC. **feyncop** can calculate the coproduct of all 2346 1PI, φ^4, 8-loop diagrams with four external legs within ten minutes.
References:

[1] McKay, B.D., Practical Graph Isomorphism, Congressus Numerantium, 30 (1981) 45–87
[2] A. Connes and D. Kreimer, Renormalization in quantum field theory and the Riemann–Hilbert problem I. The Hopf algebra structure of graphs and the main theorem, Commun. Math. Phys. 210 (1) (2000) 249–273.

1. Introduction

The purpose of this paper is to provide two tools for the computation of perturbative expansions of quantum field theory amplitudes. These expansions typically come in terms of Feynman graphs, each of them corresponding to a term in the expansion. These graphs carry Hopf algebra structures on them using the partial order provided by subgraphs. In particular the Hopf algebra structure provided by superficially divergent graphs is needed for the process of renormalization which is indispensable for the perturbative evaluation of a finite amplitude in accordance with renormalized Feynman rules.

All singularities appear in the integrand which is assigned to a Feynman graph by the Feynman rules. These integrands can be studied through the two Symanzik polynomials and the corolla polynomial for the case of gauge theories [1,2]. Short distance singularities which need to be eliminated by renormalization correspond to zeros in the first Symanzik polynomial. This basic fact underlies the utility of Hopf algebras in the study of quantum field theory.

The two tools provided here hence are a graph generator for Feynman graphs, **feyngen** and a routine which automates the Hopf algebra structure of those graphs, **feyncop**.

Additionally to the study of perturbative quantum field theories and their renormalization in general, these programs are aimed to be used as input for systematic parametric integration techniques to evaluate Feynman amplitudes [1,3]. These upcoming techniques initiated an interest in relatively large loop order Feynman diagrams. Traditional programs like **QGRAF** [4] are designed to generate low loop order diagrams and are thereby insufficient for these applications. Therefore, both programs use the established **nauty** package, described in [5], to ensure high performance graph generation at high loop orders.

feyngen is capable of generating φ^k-theory, QED and Yang–Mills Feynman graphs and of filtering these graphs for the properties of connectedness, one-particle-irreducibleness, 2-vertex-connectivity and tadpole-freeness. It can handle graphs with fixed external legs as well as those without fixed external legs.

This paper is organized as follows: An introduction to the properties of the Hopf algebra of Feynman graphs is given in Sections 2 and 3. These properties were used to derive Theorem 1, an identity on the Hopf algebra, suitable to validate the coproduct computation of **feyncop**. The combinatorial proof of this theorem constitutes a simplification of the proof given in [6]. In Section 4 details to the implementation and validation of **feyngen** are laid out and in Section 5 the implementation of **feyncop** is described.

The manuals of the Feynman graph generation program **feyngen** and the coproduct computation program **feyncop** are given in Sections 6 and 7. Furthermore, conclusions are drawn and some further prospects are outlined.

2. Feynman graphs

Viewed from a graph theoretical point, Feynman graphs are edge-colored multigraphs. Some properties and notions for multigraphs are reviewed in Appendix A. For the treatment of the Hopf algebra of Feynman graphs, a Taylor-made definition as in [2] is convenient.

2.1. Definition

Let $G = (V, E)$ be a multigraph with the vertex set V and edge multiset E being disjoint unions of the sets and multisets $\Gamma^{[0]}_{\text{int}}$, $\Gamma^{[0]}_{\text{ext}}$, $\Gamma^{[1]}_{\text{int}}$ and $\Gamma^{[1]}_{\text{ext}}$, such that

$$V = \Gamma^{[0]}_{\text{int}} \cup \Gamma^{[0]}_{\text{ext}}$$

and

$$E = \Gamma^{[1]}_{\text{int}} \cup \Gamma^{[1]}_{\text{ext}}.$$

The vertices in $\Gamma^{[0]}_{\text{ext}}$ shall have valency 1 and those in $\Gamma^{[0]}_{\text{int}}$ valency ≥ 3. The edges in $\Gamma^{[1]}_{\text{int}}$ must be incident only to vertices $v \in \Gamma^{[0]}_{\text{int}}$ and those in $\Gamma^{[1]}_{\text{ext}}$ are incident to at least one vertex $v \in \Gamma^{[0]}_{\text{ext}}$. The vertices in $\Gamma^{[0]}_{\text{int}}$ are called internal vertices and those in $\Gamma^{[0]}_{\text{ext}}$ are called external or source vertices. The edges in $\Gamma^{[1]}_{\text{int}}$ are called internal edges and the ones in $\Gamma^{[1]}_{\text{ext}}$ are called external edges or legs.

Definition 1. A Feynman graph $\Gamma = (G, \text{res})$ is a pair of a multigraph G with the above properties and a coloring res, a map

$$\text{res} : \Gamma^{[1]}_{\text{int}} \cup \Gamma^{[1]}_{\text{ext}} \to \mathcal{R}_E, \quad (1)$$

which assigns a color or type from a set of allowed edge types $\mathcal{R}_E$ to every edge in G.

The map res can be extended to the internal vertices of Γ,

$$\text{res} : \Gamma^{[0]}_{\text{int}} \cup \Gamma^{[1]}_{\text{int}} \cup \Gamma^{[1]}_{\text{ext}} \to \mathcal{R}_V \cup \mathcal{R}_E, \quad (2)$$

by assigning a vertex type $r_V \in \mathcal{R}_V$ to every internal vertex $v \in \Gamma^{[0]}_{\text{int}}$, such that the vertex types are determined uniquely by the edges incident to v. The elements $r \in \mathcal{R}_V \cup \mathcal{R}_E$ are called the allowed residue types of the theory under inspection. Note that internal vertices can be promoted to corollas of equal valence by dividing adjacent internal or external edges into suitable half-edges and assigning a corolla of valence one to an external vertex.

Generally, the Feynman rules restrict the sets $\mathcal{R}_V$ and $\mathcal{R}_E$, such that only Feynman graphs with certain vertex and edge types are