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Variable-boostable chaotic flows

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ABSTRACT

A new regime of chaotic flows is explored in which one of the variables has the freedom of offset boosting. By a single introduced constant, the DC offset of the variable can be boosted to any level, and therefore the variable can switch between a bipolar signal and a unipolar signal according to the constant. This regime of chaotic flows is convenient for chaos applications since it can reduce the number of components required for signal conditioning. Offset boosting can be combined with amplitude control to achieve the full range of linear transformations of the signal. The symmetry of the variable-boostable system may be destroyed by the new introduced boosting controller; however, a different symmetry is obtained that preserves any existing multistability.

1. Introduction

Amplitude control is an important issue in engineering applications for optimizing the amplitude [1–7] and achieving stability [6–9]. Partial amplitude control changes the amplitude of some of the variables using a partial controller [1], while a total amplitude controller adjusts the amplitude of all the variables simultaneously [2–6]. Moreover, it is often useful to transform a bipolar signal to a unipolar signal or vice versa [10–12]. For example, an ADC chip usually needs a non-negative analog signal as the input signal, which thus demands a unipolar signal. Unipolar signals are easier to transmit in directly-coupled integrated circuits. Many of the signals from physical sensors are unipolar. However, bipolar signals have lower levels of DC component, which reduces the power requirements and reduces the attenuation at high voltage and is thus good for signal transmission.

A natural question is how best to modify a bipolar signal in a differential chaotic system to make it unipolar or vice versa. A simple capacitor can transform a unipolar signal to a bipolar one since the DC component is blocked by the capacitor. But the capacitance must be large enough to have negligible reactance compared with the load. Alternately, a single-supply operational amplifier can offset the voltage and transform a unipolar signal to a bipolar one. Chaotic signals are broadband with low-frequency components, which complicates the process. A large capacitor or a broadband adder circuit is needed to achieve the transformation between the unipolar signal and a bipolar one.

In Section 2, we consider examples of chaotic flows that provide offset boosting by a single constant in the governing equations. In Section 3, we combine offset boosting with amplitude control to achieve a wide range of signals without

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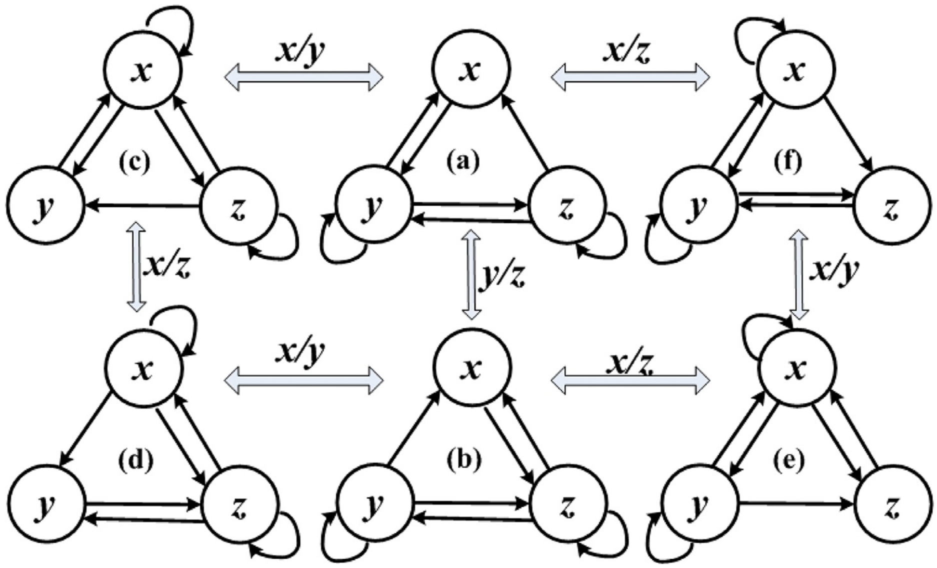

Fig. 1. The inner structure of the variable-boostable chaotic flows.

affecting their dynamical properties such as their power spectra and Lyapunov exponents. In Section 4, we show that offset boosting in symmetric systems can preserve the bistability, and the offset-boosted symmetric system can also give a symmetric pair of coexisting attractors in coordinate-shifted basins of attraction. A short conclusion and discussion are given in the last section.

2. Variable-boostable chaotic flows

Since the derivative of a constant is zero, a differential equation will not change its form if a constant is added to a variable, provided that variable does not appear explicitly. For example, replacing the variable x with $x+c$ (here c is a constant) in the equation $\dot{x} = f(y, z)$ has no effect on the dynamics. Consequently, if the other equations of the system have only a single linear occurrence of the variable x, the introduction of the constant into that equation will produce an offset of the variable x and thus give the freedom to alter the chaotic signal from unipolar to bipolar or vice versa.

Definition 1.

Suppose there is a differential dynamical system, $\dot{X} = F(X)(\mathrm{X} = \left(x_1, x_2, x_3, \ldots x_{i,} \ldots\right)(i \in N)$. If $x_i = u_i + c$ is subject to the same governing equation except through introducing a single constant into one of the other equations, *i.e.*, $\dot{Y} = F(Y, c)(\mathrm{Y} = \left(x_1, x_2, x_3, \ldots u_{i,} \ldots\right)(i \in N))$, then the system is a variable-boostable system since it has the freedom for offset boosting the variable x_i. Setting x_i with $x_i + c$ will introduce in the x_i variable a new constant c which will change the average value of the variable x_i. Thus the transformation is convenient to offset the bipolar signal x_i by a unipolar DC voltage in the circuit, or vice versa.

For a three-dimensional system, any of the variables (x, y, z) can be boosted by a constant. Consequently, there are three cases for variable boosting. To limit the complexity of the examples, we consider only quadratic nonlinearities. For such a dynamic system, if the variable x needs to be boosted by a constant, the equation can be in the form of Eq. (1), where the extra constant is introduced in the dimension of z.

$$\begin{cases} \dot{x} = a_1 y + a_2 z + a_3 y^2 + a_4 z^2 + a_5 yz + a_{16}, \\ \dot{y} = a_6 y + a_7 z + a_8 y^2 + a_9 z^2 + a_{10} yz + a_{17}, \\ \dot{z} = a_{11} y + a_{12} z + a_{13} y^2 + a_{14} z^2 + a_{15} yz + a_{18} x. \end{cases} \tag{1}$$

Similarly, the variables y and z can be boosted by a new introduced constant in other dimensions. However, all such cases can be written in the form of Eq. (1) without loss of generality through a simple transformation of variables. This can be confirmed from the topological structure as shown in Fig. 1. The variable without self-feedback can receive offset boosting control from the variable in the arm with dual-direction connectivity. For example, in the structure (b), the variable x has no self-feedback; the variable x influences the dynamics only by the dimension of z leading to a dual-direction connection. Therefore, the variable x can obtain offset boosting from the dimension of z. All these cases can be transformed from any of the other cases by the variable substitutions marked in Fig. 1.

Some cases conforming to the above topological structures have been given by Sprott [13,15,16]. For electrical circuit implementation, a standard jerk equation usually leads to a compact circuit topology. Therefore, for the convenience of