Optik 127 (2016) 10643–10652

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Analysis and circuit simulation of a novel nonlinear fractional incommensurate order financial system

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ARTICLE INFO

Article history:
Received 28 June 2016
Accepted 28 August 2016

Keywords:
Fractional incommensurate order systems
Financial system
Chaotic systems
Circuit realization

ABSTRACT

In this paper, we consider a fractional incommensurate order financial system which is a generalized form of the financial system recently reported in the literature. Many interesting dynamic behaviors can be seen in fractional incommensurate order financial system e.g. chaotic motions, periodic motions and fixed points. Phase portraits and time histories of fractional incommensurate order financial system are exhibited. Adopting the largest Lyapunov exponent criteria, we find that the system yields chaos at the lowest order of 2.10. Intermittent chaotic behavior can be seen in the fractional-order financial system. In order to confirm the feasibility of the theoretical model, Cadence OrCAD package is used to design an electronic circuit to emulate the behavior of the novel nonlinear fractional order finance system.

1. Introduction

Recently, a new scientific topic known as "Econophysics" has turn to an attractive field of research to study the complex dynamics of real economic systems. Aiming at this goal, researchers are striving to explore main features of economic theory such as irregular and erratic economic fluctuations, overlapping waves of economic development and structural changes. The economist takes into account a model composed of only endogenous variables, in which the model has a simple behavior. Then the economist augments the model by considering exogenous shock variables which is due to the random external events e.g. weather variables, political events, and other human factors. In contrast to the above mentioned viewpoint, chaos explains the complexity observed in economic series. Since chaos can represent radical changes of perspective on business cycles [1], it has gained a significant importance in economics.

In a definite system, chaos is an inherent randomness which is not caused by system external disturbances. Phase portraits with complex patterns and positive Lyapunov exponents which are the main features of deterministic chaos, can be seen in many economic data. In order to study complex economic dynamics, many chaotic models have been presented in recent studies [2–10].

In order to investigate the integrals and derivatives of arbitrary order, fractional calculus is becoming more attractive tool in various fields of physics, economics and engineering in recent years [11–15]. Since real phenomena are generally fractional, fractional order derivatives can be used to describe different nonlinear phenomena more accurately than integer order derivatives [15–18]. Fractional order modeling tools result in a more accurate description and a deeper insight into natural and man-made processes that are underlying a long-range memory behavior more accurately than the integer

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http://dx.doi.org/10.1016/j.ijleo.2016.08.098

order modeling tools. Thus, fractional calculus is applied in various problems related to electronic circuits, viscoelasticity, chemistry, electrical conductance of biological systems, modeling of neurons, psychological, life sciences, market dynamics, Chen chaotic and hyper chaotic systems, chaos and hyper chaos control and synchronization of fractional order chaotic and hyper chaotic systems [19–24].

Due to correlations with the longest time scale in the market, financial variables can have very long memory [5,25]. In the other hand, all future fluctuations in financial variables are dependent on current and previous fluctuations. Since financial fractional nonlinear model has simultaneously memory and chaos, it makes us motivated to use it as a modeling tool to describe financial systems.

Based on the above discussion, we use the fractional order model to modify the system in [26]. The chaotic behavior of the system are explored via phase portraits of the system state variables and largest Lyapunov exponent criteria. Using Cadence OrCAD package, an electronic circuit is realized to emulate the proposed fractional nonlinear finance system.

The organization of this paper is as follows: The proposed system is introduced in Section 2. Analysis of the novel chaotic fractional order finance system are studied in Section 3. Circuit realization and its OrCAD simulation of the model are presented in Section 4. Section 5 concludes the paper.

2. Novel chaotic fractional incommensurate order financial system

In this section, we present basic definitions and concepts of the fractional calculus and introduce a novel nonlinear fractional incommensurate order finance system.

There are several definitions of fractional differential operator. Since Caputo fractional differential operator describes initial conditions of fractional order dynamic systems similar to initial conditions of integer order systems, we use it for definition of the novel nonlinear fractional order finance system. The Caputo fractional derivative of function $f(t)$ with respect to time is described as [15]:

$$D^q = \begin{cases} \dfrac{1}{\Gamma(n-q)} \displaystyle\int_0^t (t-\tau)^{n-q-1} f^{(n)}(\tau)d\tau, & m-1 < q < m \\ \dfrac{d^n}{dt^n} f(t), \quad q = n, \end{cases} \tag{1}$$

where q is the order of fractional derivative, m is the lowest integer which is not less than q, and Γ is the Gamma function,

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt. \tag{2}$$

A dynamic model of finance which is composed of three first-order differential equations have been reported in [26]. The time variation of three state variables is included in the model, namely: the interest rate which is denoted by x_1, the investment demand which is denoted by x_2 and the price index which is denoted by x_3. The x_1 is mainly influenced by two factors. First, structural adjustment from good prices, and second, contradictions from the investment market, i.e. the surplus between investment and savings. The changing rate of x_2 is influenced by three factors. First, rate of investment, second, cost of investment and third, interest rates. The x_3 is mainly influenced by two factors: first, inflation rates, and second, a contradiction between supply and demand in commercial markets. The following set of differential equations describe nonlinear finance chaotic system [26]:

$$\begin{cases} \dfrac{dx_1}{dt} = x_3 + (x_2 - a)x_1 \\ \dfrac{dx_2}{dt} = 1 - bx_2 - |x_1| \\ \dfrac{dx_3}{dt} = -x_1 - cx_3, \end{cases} \tag{3}$$

where a denotes the saving amount, b denotes the cost per investment and c denotes the elasticity of demand of commercial markets. It is clear that all three parameters have a positive value ($a > 0$, $b > 0$, $c > 0$).

Here, we propose an extended fractional incommensurate-order form of the system (3) which takes the following form:

$$\begin{cases} \dfrac{d^{q_1}x_1}{dt^{q_1}} = x_3 + (x_2 - a)x_1 \\ \dfrac{d^{q_2}x_2}{dt^{q_2}} = 1 - bx_2 - |x_1| \\ \dfrac{d^{q_3}x_3}{dt^{q_3}} = -x_1 - cx_3. \end{cases} \tag{4}$$

where $\mathbf{x} = (x_1, x_2, x_3)^T \in R^3$ is the state vector, $q_i \in [0, 1]$ is the fractional derivative order for $i = 1, 2, 3$. a, b, c are constant positive parameters of the system.