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Auto-Bäcklund transformation for some nonlinear partial differential equation

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ABSTRACT

In this paper, we apply auto-Bäcklund transformation for Sharma-Tasso-Olver (STO) equation and fourth order equation of the Burgers hierarchy. Auto-Bäcklund transformation was developed as a direct and simple method to obtain solutions of nonlinear partial differential equations by Fan.We obtain solitary wave solutions of these equations.The numerical simulations are added to these obtained solutions

1. Introduction

Nonlinear partial differential equations (NPDEs) have a significant place in applied mathematics and physics. These equations are mathematical models of physical phenomenon that arise in engineering, chemistry, biology, mechanics and physics. It is very significant to obtain information about the solutions of mathematical models. To better understand the mechanisms of the mathematical models, it is necessary to solve these equations. Thus, it has an important place to obtain the analytic solutions of nonlinear differential equations in applied sciences. Recently, it has become attractive solving these equations. Therefore, some methods have been developed by sciences. Some of these methods are: Hirota method [1], Bäcklund transformation [2], Cole-hopf transformation method [3], Generalized Miura Transformation [4], inverse scattering method [5], Darboux transformation [6], Painleve method [7], homogeneous balance method [8], similarity reduction method [9] and sine cosine method [10].

In this paper, we investigate solitary wave solutions of STO equation [11] and fourth order equation of the Burgers hierarchy [12] by using auto-Bäcklund transformation [13]. STO equation and fourth order equation of the Burgers hierarchy are defined as follows, respectively;

$$u_t + 3u_x^2 + 3uu_{xx} + 3u^2u_x + u_{xxx} = 0, \tag{1}$$

and

$$u_t + 10u_xu_{xx} + 4uu_{xxx} + 12uu_x^2 + 6u^2u_{xx} + 4u^3u_x + u_{xxxx} = 0, \tag{2}$$

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STO equation was first developed as an example of odd members of the Burgers hierarchy by extending the “linearization” obtained through the Cole-Hopf ansatz to equations involving as highest derivatives odd space derivatives [14]. Wazwaz [15] obtained solitons and kinks solutions for STO equation by using the tanh method, the extended tanh method. Ugurlu and Kaya [16] investigated some exact solutions of STO equation by using the improved tanh function method. Jawad et al. [17] obtained exact solutions for STO equation by using modified simple equation method. Shang et al. [18] presented exact explicit travelling wave solutions of STO equation by using the extended hyperbolic function method.

Burgers equation is used for mathematical modeling of fluid mechanics, nonlinear acoustic gas dynamics, traffic flow, the theory of shock waves, turbulence problems. Different types of Burgers equation are given in literature such as inviscid Burgers’ equation, viscous Burgers’ equation, Burgers-like equation and coupled Burger’s equations. Eq. (2) is called the fourth order equation of the Burgers hierarchy. Many mathematicians investigated different forms of Burgers’ equation [19–21]. Wazwaz [12] found multiple kink solutions and multiple singular kink solutions of Eq. (2).

2. Two applications of auto-Bäcklund transformation

Example 1. In accordance with the idea of improved HB [22], we investigate for Bäcklund transformation of Eq. (1). When balancing u^2u_x with u_{xxx} then gives $M = 1$. Hence, we can write

$$u = \frac{\partial}{\partial x} f(w) + u_0 = f'w_x + u_0, \tag{3}$$

where$f = f(w)$, $w = w(x, t)$, $u_0 = u_0(x, t)$. Here$f = f(w)$ and $w = w(x, t)$ are uncertain functions,$u$and u_0 are two solutions of Eq. (1). Substituting Eq. (3) into Eq. (1), we obtain

$$\begin{aligned}
&u_t = w_t w_x f'' + w_{xt} f' + (u_0)_t \\
&3u_x^2 = 3(f'')^2 w_x^4 + 6f'f''w_x^2 w_{xx} + 6f''w_x^2(u_0)_x + 3(f')^2 w_{xx}^2 + 6f'w_{xx}(u_0)_x + 3(u_0)_x^2 \\
&3uu_{xx} = 3f'f'''w_x^4 + 9f'f''w_x^2 w_{xx} + 3(f')^2 w_x w_{xxx} + 3f'w_x(u_0)_{xx} + 3f'''w_x^3 u_0 + 9f''w_x w_{xx} u_0 \\
&+3f'w_{xxx}u_0 + 3u_0(u_0)_{xx} \\
&3u^2u_x = 3(f')^2 f'' w_x^4 + 3(f')^3 w_x^2 w_{xx} + 3(f')^2 w_x^2 (u_0)_x + 6f'f''w_x^3 u_0 + 6(f')^2 w_x w_{xx} u_0 \\
&+6f'w_x u_0(u_0)_x + 3f''w_x^2 u_0^2 + 3f'w_{xx}u_0^2 + 3u_0^2(u_0)_x \\
&u_{xxx} = f^{(4)} w_x^4 + 6f'''w_x^2 w_{xx} + 3f''w_{xx}^2 + 4f''w_x w_{xxx} + f'w_{xxxx} + (u_0)_{xxx} \\
&\text{and} \\
&(f^{(4)} + 3(f'')^2 + 3f'f''' + 3(f')^2 f'')w_x^4 + (6f'''w_x^2 w_{xx} + 15f'f''w_x^2 w_{xx} + 3(f')^3 w_x^2 w_{xx} + 3f'''w_x^3 u_0 + 6f'f''w_x^3 u_0) \\
&+\left(\begin{array}{l} w_x w_t f'' + 3f''w_{xx}^2 + 4f''w_x w_{xxx} + 6f''w_x^2(u_0)_x + 3(f')^2 w_{xx}^2 + 3(f')^2 w_x w_{xxx} + 9f''w_x w_{xx} u_0 \\ +3(f')^2 w_x^2 (u_0)_x + 6(f')^2 w_x w_{xx} u_0 + 3f''w_x^2 u_0^2 \end{array}\right) \\
&+(f'w_{xt} + f'w_{xxxx} + 6f'w_{xx}(u_0)_x + 3f'w_x(u_0)_{xx} + 3f'w_{xxx}u_0 + 6f'w_x u_0(u_0)_x + 3f'w_{xx}u_0^2) = 0
\end{aligned} \tag{4}$$

Setting the coefficients of w_x^4 in Eq. (4) to zero, we have ordinary differential equation (ODE)

$$f^{(4)} + 3(f'')^2 + 3f'f''' + 3(f')^2 f'' = 0$$

where

$$f = \ln w \tag{5}$$

there by from Eq. (5) it holds that

$$(f')^2 = -f'' \ , \quad f'f'' = -\frac{1}{2} f''' \ , \quad (f')^3 = \frac{1}{2} f''' \tag{6}$$

By using Eqs. (6) and (4) can be written as the sum of some terms with f' and f'' setting their coefficients to zero will lead to

$$w_x(w_t + w_{xxx} + 3w_x(u_0)_x + 3w_{xx}u_0 + 3w_x u_0^2) = 0.$$

$$\frac{\partial}{\partial x}(w_t + w_{xxx} + 3w_x(u_0)_x + 3w_{xx}u_0 + 3w_x u_0^2) = 0$$

Above equation is satisfied provided that

$$w_t + w_{xxx} + 3w_x(u_0)_x + 3w_{xx}u_0 + 3w_x u_0^2 \tag{7}$$