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Affine registration of point clouds based on point-to-plane approach

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Abstract

The problem of aligning of 3D point data is the known registration task. The most popular registration algorithm is the Iterative Closest Point (ICP). This paper proposes a new algorithm for affine registration of point clouds by incorporating the affine transformation into the point-to-plane ICP algorithm. At each iterative step of the algorithm, a closed-form solution for the affine transformation is derived.

Keywords: Affine registration; Iterative closest points (ICP); non-rigid ICP; point-to-plane; surface reconstruction

1. Introduction

The ICP (Iterative Closest Point) algorithm has become the dominant method for aligning of three-dimensional models based purely on geometry [1,2]. The algorithm is widely used for registering the outputs of 3D scanners and optics systems for 3D scene reconstruction [3-5]. The ICP algorithm starts with two clouds and an initial guess for their relative rigid-body transformation, and then it iteratively refines the transformation by generating pairs of corresponding points on the meshes. The initial alignment may be done by different methods, such as tracking scanner position, identification and indexing of surface features [6,7], "spin-image" surface signatures [8],

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computing principal axes of scans [9], exhaustive search for corresponding points [10,11], or in an interactive manner. Since introduction of the ICP [1,2], many variations of the algorithm have been introduced on the base of the ICP concept.

The ICP algorithm consists of two main stages:

1. Searching of corresponding points (pairs) in two clouds.
2. Minimizing the error metric (variational subproblem of the ICP).

The step 2 is a key-point of the ICP [15]. There are two main approaches for the selecting the error metric. The point-to-point method [2] uses the distance between corresponding points in two clouds. The point-to-plane method [1] utilizes the distance between a point in the first cloud and a tangent plane to the second cloud. A solution of the error-minimizing problem is based on the affine transformation that yields the best alignment between points of the two clouds. In the orthogonal case, the closed-form point-to-point solution was proposed by Horn [12,13]. The computational complexity of the solution is linear with respect to the number of points.

The traditional ICP algorithm is fast and accurate for the rigid registration between two point clouds but it is unable to handle with the affine case. A modified algorithm of the standard ICP for composition of scaling, rotation, and translation was proposed [14]. A generalized ICP version for an arbitrary affine transformation was suggested [15,16]. These algorithms are based on the point-to-point approach. A closed-form solution to the point-to-point problem was derived [18-20]. A closed-form solution to the point-to-plane case for orthogonal transformations is an open problem. Instead, iterative methods based on the linear least-squares optimization for small angles are often used [17].

In the paper, we propose a closed-form solution to the point-to-plane problem for arbitrary affine transformation. The algorithm yields precise solutions to the variational subproblem of the ICP.

2. Formulation of the variational problem

Let $P = \{p^0, \ldots, p^{k-1}\}$ be a source point cloud and $Q = \{q^0, \ldots, q^{k-1}\}$ be a destination point cloud in $\mathbb{R}^3$. Suppose that the relationship between points in P and Q is given in such a manner that for each point p_i exists a corresponding point q_i. The ICP algorithm is commonly considered as a geometrical transformation for rigid objects mapping P to Q:

$$Rp_i + t, \tag{1}$$

where R is a rotation matrix, t is a translation vector, $i = 0, \ldots, n-1$. The S-ICP algorithm uses a slightly different geometrical transformation given as

$$RSp_i + t, \tag{2}$$

where S is a scaling matrix.

The group of affine transformations in the dimension of three has 12 generators. It means that the affine transformation is a function of 12 variables. Let us consider the ICP variational problem for an arbitrary affine transformation in the point-to-plane case. Denote by $S(Q)$ a surface, that is built on the cloud Q, by $T(q^i)$ denote a tangent plane of $S(Q)$ in point q^i. Let $J(A,T)$ be the following function:

$$J(A) = \sum_{i=1}^{n} (< Ap^i - q^i, n^i >)^2, \tag{3}$$

where <·,·> denotes the inner product, A is a matrix of an affine transformation in the homogenous coordinates:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & t_1 \\ a_{21} & a_{21} & a_{21} & t_2 \\ a_{31} & a_{31} & a_{31} & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

p^i is a point from the cloud P, n^i is the unitary normal for $T(q^i)$: