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Calculation of Maximum Safe Temperature of Fuel's Heating in a Porous Medium

Elena Shchepakina[a,*]

[a] *Samara National Research University, 34, Moskovskoye shosse, Samara 443086, Russia*

Abstract

The paper is devoted to the investigation of an autoignition of liquid fuel due to leaking into insulation porous material surrounding a pipeline. Application of the geometric theory of singular perturbations allow us to describe all possible scenarios of the process, to reveal the critical condition for the autoignition, and to calculate the maximal safe temperature of fuel's heating.

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Keywords: autoignition, critical phenomenon, maximum safe temperature, canard, integral manifold, delayed loss of stability

1. Introduction

The work is related to the study of the features of the critical autoignition phenomena of combustible fluids in porous medium [1, 2]. The fuel's autoignition is usually caused by a leaking of a combustible liquid into insulation material surrounding a hot pipe. Leakage of flammable liquids into isolation layer during a transportation or technological process can lead to spontaneous fire of the fuel. Since the insulation is a porous medium containing an oxidizer, the exothermic oxidation reaction can lead to a substantial self-heating of the fuel and its ignition and, as a result, to destruction of the pipeline. The possible depletion of oxygen or its diffusion into porous structure and transport of the liquid or its vapour within the insulation are all ignored. In the conditions of a spatially homogeneous approach the dimensionless model has the form [1]

$$\begin{aligned} \frac{du}{dt} &= QK_1 x e^{-1/u} - (u - u_a) - Q_c K_2 x e^{-\beta_e/u}, \\ \frac{dx}{dt} &= -K_2 x e^{-\beta_e/u} - K_1 x e^{-1/u}. \end{aligned} \tag{1}$$

Here, u is a dimensionless temperature of the reactant phase; the dimensionless concentration x represents the mass fraction of combustible liquid present in the porous material; the dimensionless parameters Q and K_1 characterize the heat of reaction and the reaction frequency, respectively, for the exothermic oxidation reaction, while Q_c and K_2

* Corresponding author. Tel.: +7-846-332-5786; fax: +7-846-267-4370.
E-mail address: shchepakina@ssau.ru

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are the similar terms for the endothermic evaporation reaction; β_e is the ratio of the enthalpy of vaporization to the activation energy of the oxidation reaction; u_a is ambient temperature.

System (1) may be written in more convenient form [3]

$$\begin{aligned} \varepsilon\dot{\theta} &= \mu x \exp\left(\frac{\theta}{1+\beta\theta}\right) - (\theta-\theta_a) - \nu x \exp\left(\frac{\beta_e\theta}{1+\beta\theta}\right), \\ \dot{x} &= -ax \exp\left(\frac{\beta_e\theta}{1+\beta\theta}\right) - K_1 x \exp\left(\frac{\theta}{1+\beta\theta}\right), \end{aligned} \tag{2}$$

where

$$u = \theta\beta^2+\beta,\ \ \beta = u(0),\ \ t = \tau\exp\left(\frac{1}{\beta}\right),\ \ \varepsilon = \exp\left(-\frac{1}{\beta}\right),\ \ \theta_a = \frac{u_a-\beta}{\beta^2},$$

$$a = K_2\exp\left(\frac{1-\beta_e}{\beta}\right),\ \ \mu = \frac{QK_1}{\beta^2}\exp\left(-\frac{1}{\beta}\right),\ \ \nu = \frac{Q_cK_2}{\beta^2}\exp\left(-\frac{\beta_e}{\beta}\right).$$

System (2) is singularly perturbed due to the smallness of the parameter ε for typical combustible liquids. The chemically relevant phase space Ω of system (2) is defined by $\Omega = \{x \geq 0,\ \ \theta \geq -1/\beta\}$.

In [2] system (1) was investigated numerically under the quasi-steady-state assumption that corresponds to the case $\varepsilon = 0$ in system (2). For the majority of applied problems that don't meet necessary conditions for bifurcations this approach gives acceptable results. The quasi–steady–state assumption leads to the significant simplifications in the calculations, allows, nevertheless, to determine the main features of the solutions' dynamics of the full system for sufficiently small ε. However, the detailed study of the critical phenomena is possible with taking into account the small perturbations. In such cases, the application of geometric and asymptotic approaches for qualitative analysis is useful. In present paper the theory of integral manifolds for singularly perturbed systems is used as a research tool.

A qualitative investigation of (2) with $\varepsilon \neq 0$ was carried out in [3, 4]. It was revealed that the critical phenomena in this dynamical model is linked to a canard's existence [5–10]. It was shown that the critical regime corresponds to the borderline of safe processes in the sense that it is a kind of a watershed between the explosion regimes and safe combustion modes. The goal of this paper is to determine the value of the maximal temperature during the critical regime as a maximal temperature of safe heating.

2. Analysis of the Possible Modes

System (2) has the stable steady state $\theta = \theta_a$, $x = 0$, reachable in final time. If we put $\varepsilon = 0$ in (2) we get the degenerate equation

$$0 = \mu x\exp\left(\frac{\theta}{1+\beta\theta}\right) - (\theta-\theta_a) - \nu x\exp\left(\frac{\beta_e\theta}{1+\beta\theta}\right) = F(x,\theta), \tag{3}$$

which describes *the slow curve* S of (2) (see, for example, [10, 11]).

The subset S^s (S^u) of S with

$$\frac{\partial}{\partial\theta}F(x,\theta) < 0\ \ (>0)$$

is called *the stable or attractive (unstable or repulsive) part of* S. A point A on S in which $\partial F/\partial\theta = 0$ is called *the jump or turning point*. Stable and unstable parts of the slow curve are zeroth order approximations of corresponding stable and unstable slow invariant manifolds. The invariant manifolds lie in an ε-neighborhood of the slow curve, except near jump or turning points (see [11] and references therein).

The slow curve S has an asymptote $\theta = \theta_\infty$, where

$$\theta_\infty = \frac{1}{\beta_e - 1 - \beta\ln(\mu/\nu)}\ln\frac{\mu}{\nu},$$

and intersects the axis $O\theta$ in the point with $\theta = \theta_0$. The shape of the curve S varies with the relation between values of the parameters ν, μ and β_e, which leads to a change in qualitative behavior of the system. So, if we change the value