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Modeling of Nonstationary Distributed Processes on the Basis of Multidimensional Time Series

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Abstract

A method for identifying the equations of mathematical physics describing the dynamics of spatially-distributed processes on the basis of experimental multidimensional time series is proposed. The method includes the LSM (Least square method) estimates of the parameters of multidimensional autoregression and the construction of versions of systems of algebraic equations connecting the estimates of autoregression and the parameters of the corresponding differential equations. The system of algebraic equations satisfied by the obtained estimates determines the structure of the model and the corresponding values of the parameters of differential equation. A numerical example of identifying the processes of changing the temperature of atmospheric air is given.

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Keywords: Partial Differential Equations; Structural Identification; Parametric Identification; Multidimensional Autoregression; Statistical Hypotheses

1. Introduction

Usually the modeling of physical processes is based on a combination of two approaches: mechanistic (axiomatic) and statistical. The mechanistic approach assumes that the mechanisms of the functioning of the process are known and can be described by the form of the corresponding systems of differential equations [1].

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Nomenclature

v	advection velocity
D	diffusion coefficient
x_i^t	level of time series (observed value)
$\hat{x}_i^t$	value obtained as a result of solution of the difference equations
x_i^t	value obtained as a result of modeling of the time series

The statistical approach is based on the construction of heuristic models of the behavior of the process using the results of observation of process' variables. At the same time the linear stochastic models of autoregression, namely the models of sliding, are widely used [2, 3]. If there is not enough information to make the decisions about the exact mechanism of the process (i.e., it is possible to construct the variants (structural and parametric) of the systems of differential equations), the additional application of the statistical approach can greatly reduce the initial uncertainty and increase the adequacy of the model. We consider a wide class of spatially-distributed dynamical systems, where the diffusion processes, advection processes or their combination take place. The source of information on the behavior of the system is the measured data of some characteristics x_i^t at successive instants of time $t = 0,1,$ in the nodes of a one-dimensional spatial regular grid $i = 0,1, \; ,n$, i.e., the multidimensional time series. Consideration of a one-dimensional grid does not limit further research, but it avoids the cumbersome constructions typical for two- and three-dimensional spaces.

The problem is to develop the algorithms for structural and parametric identification of a mechanistic model with constant coefficients by the observed values x_i^t. Let us suppose that diffusion and advection processes can occur in the system under consideration. The general form of the corresponding equation is

$$\frac{\partial x}{\partial t} + v\frac{\partial x}{\partial l} = D\frac{\partial^2 x}{\partial l^2}, \quad x(0,l) = \varphi(l), \tag{1}$$

where v is the advection velocity, D is the diffusion coefficient, l is the spatial coordinate.

There are no sufficient grounds to assert that the behavior of the system is determined by one of these processes, or both of them simultaneously. The parameters in (1), that are considered as constants obtained as a result of averaging over the space and time are also unknown. It is necessary to obtain the information on the structure and parameters of the system under consideration using the observed data x_i^t.

2. Difference schemes for versions of equations in mechanistic model

In order to solve the problem, we will get the explicit four-point difference schemes for the equations of each of the variants of the structure of processes:

diffusion and advection $\dfrac{x_i^{t+1} - x_i^t}{\Delta t} + v\dfrac{x_{i+1}^t - x_{i-1}^t}{2\Delta l} = D\dfrac{x_{i+1}^t - 2x_i^t + x_{i-1}^t}{\Delta l^2}$,

$$x_i^{t+1} = (b_1 + b_2)x_{i-1}^t + (1-2b_2)x_i^t + (b_2 - b_1)x_{i+1}^t; \quad b_1 = \frac{v\Delta t}{2\Delta l}; \quad b_2 = \frac{D\Delta t}{\Delta l^2}; \tag{2}$$

diffusion in a stationary medium $\dfrac{x_i^{t+1} - x_i^t}{\Delta t} = D\dfrac{x_{i+1}^t - 2x_i^t + x_{i-1}^t}{\Delta l^2}$,

$$x_i^{t+1} = b_2 x_{i-1}^t + (1-2b_2)x_i^t + b_2 x_{i+1}^t; \tag{3}$$

advection $\dfrac{x_i^{t+1} - x_i^t}{\Delta t} + v\dfrac{x_{i+1}^t - x_{i-1}^t}{2\Delta l} = 0$,

$$x_i^{t+1} = b_1 x_{i-1}^t + x_i^t - b_1 x_{i+1}^t. \tag{4}$$