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Decomposition of flexible joint robot mathematical model

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Abstract

The mathematical model of the manipulator with flexible joints is investigated under the condition of weak dissipation. The method of asymptotic decomposition is used to reduce dimension and to simplify the structure of the model. The decoupling coordinate transformation is constructed as an asymptotic series. It converts original multirate system to block triangular form with independent slow subsystem.

Keywords: mathematical model; manipulator; decomposition; integral manifolds; asymptotic methods

1. Manipulator Model

We consider the dynamic model of n-links manipulator with flexible joints. The flexibility of each joint is represented by torsion spring with sufficiently large elastic coefficient. The dynamics of robot is described by the equations [1–3]

$$\begin{aligned} &D(q_1)\ddot{q}_1 + c(q_1,\dot{q}_1) + K(q_1 - q_2) + B(\dot{q}_1 - \dot{q}_2) = 0, \\ &J\ddot{q}_2 - K(q_1 - q_2) - B(\dot{q}_1 - \dot{q}_2) = u, \end{aligned} \tag{1}$$

where the coordinates of vectors $q_1 \in R^n$ and $q_2 \in R^n$ are angles which characterize links and rotors positions respectively, $D(q_1)$ is inertia matrix due to the links, J is diagonal inertia matrix of drive rotors, vector $c(q_1, \dot{q}_1)$ is determined by coriolis, centrifugal and gravitation components, $K = k\, diag(\widetilde{K}_1, \ldots, \widetilde{K}_n)$ is diagonal matrix of elastic coefficients, $B = diag(B_1, \ldots, B_n)$ is diagonal matrix of damping ratios, u is the unit torque.

Let $\mu = 1/k$ be a small positive parameter. Putting $q = q_1,\ z = k(q_1 - q_2)$ gives us the system

$$\begin{aligned} \ddot{q} &= a_1(q,\dot{q}) + A_1(q)z + \mu A_3(q)\dot{z}, \\ \mu\ddot{z} &= a_2(q,\dot{q}) + A_2(q)z + \mu A_4(q)\dot{z} + M_2 u, \end{aligned} \tag{2}$$

where

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$$a_1(q,\dot q) = a_2(q,\dot q) = -D^{-1}(q)c(q,\dot q), \quad A_1(q) = -D^{-1}(q)\widetilde{K}, \quad A_3(q) = -D^{-1}(q)B,$$
$$A_2(q) = -(D^{-1}(q) + J^{-1})\widetilde{K}, \quad A_4(q) = -(D^{-1}(q) + J^{-1})B, \quad M_2 = -J^{-1}.$$

In [1,2] the concept of combine control was used. It was supposed that $B_j = \widetilde{B}_j/\mu$ or $B_j = \widetilde{B}_j/\sqrt{\mu}$, where $\widetilde{B}_j = O(1)$. This corresponds to sufficiently high dissipation. This assumption guarantees the fulfillment of Tichonov's theorem condition about asymptotic stability of the so-called boundary layer system. Let us suppose that $B_j = O(1)$. Then condition of this theorem is not fulfilled. This makes it difficult to apply classic asymptotic methods of analysis.

2. Decomposition

One of the approaches, which allows to reduce the complex multirate dynamic systems, is the asymptotic decomposition method based on the theory of integral manifolds [4–12]. This method combines elements of geometric and asymptotic methods of analysis. An integral manifold approach is used in [1,2] to solve the problems of control by constructing the reduced model of manipulator with flexible joint under the condition of sufficiently high dissipation. The method of integral manifolds was developed in [12,13] for some classes of quasi-oscillating systems. The results are used to solve the problems of control and observation for robots [14–16].

The main purpose of this research is the construction of a coordinate transformation which converts original multirate system to block triangular form with independent slow subsystem and fast subsystem which describes damped high-frequency oscillations. This will make it possible to use the slow subsystem as a simplified model of the manipulator.

Let $\varepsilon = \sqrt{\mu}$ be the new small positive parameter, $x_1 = q,\ x_2 = \dot q,\ y_1 = z,\ y_2 = \varepsilon\dot z.$

The system (2) can be rewritten as

$$\begin{aligned} \dot x &= f(x) + F(x,\varepsilon)y, \\ \varepsilon\dot y &= p(x) + P(x,\varepsilon)y + Mu(t,x,\varepsilon), \end{aligned} \tag{3}$$

where

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad F(x,\varepsilon) = \begin{pmatrix} 0 & 0 \\ A_1(x_1) & \varepsilon A_3(x_1) \end{pmatrix}, \quad f(x) = \begin{pmatrix} x_2 \\ a_1(x_1,x_2) \end{pmatrix},$$
$$p(x) = \begin{pmatrix} 0 \\ a_2(x_1,x_2) \end{pmatrix}, \quad M = \begin{pmatrix} 0 \\ M_2 \end{pmatrix}, \quad P(x,\varepsilon) = \begin{pmatrix} 0 & I \\ A_2(x_1) & \varepsilon A_4(x_1) \end{pmatrix}.$$

Let the control law $u(t,x,\varepsilon)$ be represented as

$$u(t,x,\varepsilon) = u_0(t,x) + \varepsilon u_1(t,x) + \varepsilon^2 u_2(t,x,\varepsilon) + \ldots$$

The slow integral manifold of the system (3) is

$$y = h(t,x,\varepsilon). \tag{4}$$

The conditions of the existence of such integral manifold for quasi-oscillatory systems are studied in [12,13].

The vector–function $y = h(t,x,\varepsilon)$ may be constructed with any degree of accuracy as an asymptotic power series of the small parameter ε

$$h = h(t,x,\varepsilon) = h^{(0)}(t,x) + \varepsilon h^{(1)}(t,x) + \varepsilon^2 h^{(2)}(t,x) + \ldots \tag{5}$$

from the equation

$$\varepsilon(\frac{\partial h}{\partial t} + \varepsilon\frac{\partial h}{\partial x}(f(x) + F(x,\varepsilon)h) = p(x) + P(x,\varepsilon)h + Mu(t,x,\varepsilon). \tag{6}$$

Substituting (5) to (6) and equating coefficients at the same powers of ε we get

$$h^{(0)} = h^{(0)}(t,x) = -(P^{(0)}(x))^{-1}(p(x) + Mu_0(t,x)),$$