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Integral manifolds of fast-slow systems in nonholonomic mechanics

Alexander Kobrin[a], Vladimir Sobolev[b]*

[a]National Research University "Moscow Power Engineering Institute", Moscow, Russia
[b]Samara National Research University, 34 Moskovskoe Shosse, Samara 443086, Russia

Abstract

The paper is devoted to the application of methods for constructing invariant integrals of fast-slow systems to the problems of "nonholonomic mechanics".

Keywords: Invariant manifolds; singular perturbations; nonholonomic systems; Chaplygin sleigh

1. Introduction

The proposed paper is devoted to the application of methods for constructing invariant integrals of fast-slow systems [1, 2] to the problems of "nonholonomic mechanics" [3]. When using a nonholonomic model, the external force field is usually not completely specified. However, the degenerate (limiting) system is defined. For this, the "experimental material" is used on certain quasi-speeds-linear forms of generalized velocities of points of the system with coefficients that depend on generalized coordinates. The original system of equations can be extended using the behavior in the rapidly current time of solutions of the Tikhonov system [4]. Almost always there is an existence in the non-holonomic system of the first integral of the type of the law of conservation of energy and, at the same time, the presence of attractors, non-isolated equilibrium points, and so on. The point is that the non-holonomic model is the initial approximation to the slow component on the integral (invariant) manifolds of the original problem. But in the fast-slow system there is also a fast component of the solution, which, in particular, largely takes into account the dissipation, leaving the nonholonomic model only traces of its presence (sometimes, quite expressive). The

* Corresponding author. Tel.: +7-846-334-54-38.
E-mail address: hsablem@gmail.com

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classical use of a non-holonomic system of equations for the study of theoretical-mechanical problems requires its justified application at an infinite or at least an extremely long time interval: the verification of a uniform continuous time dependence on initial conditions, the construction of the first integrals, verification of the existence of an invariant measure, and so on. A judgment about this possibility under certain conditions can be provided using methods of the theory of invariant manifolds.

2. Chaplygin sleigh

As an example of the investigation of the "nonholonomic problem" by the methods of constructing invariant integral manifolds containing the slow component of the solutions of the initial fast-slow problem, we can consider the problem of S.A. Chaplygin [5].

Let x, y are the coordinates of the center of mass, which lies on the axis located in the plane of the wheel; φ is the angle of rotation of the body; h is the distance from the point of touch of the pointed wheel to the center of mass.

The kinetic energy of the body is

$$T = \frac{1}{2}m(x^2 + y^2) + \frac{1}{2}I_c\varphi^2$$

Here m is the mass and I_c is the moment of inertia.

The equations of motion, taking into account the not completely known reaction F of the underlying surface, which is orthogonal to the wheel plane, have the form

$$mx \cong -F\sin\varphi$$

$$my \cong F\cos\varphi$$

$$I_c\omega \cong -Fh$$

Replacement of variables (from the velocities of the center of mass to the quasi-growth rate, longitudinal and transverse, at the point of contact of the wheel with the surface)

$$u = x\cos\varphi + y\sin\varphi$$

$$v = -x\sin\varphi + y\cos\varphi - h\varphi$$

$$\omega = \varphi$$

gives

$$mu = m\omega(v + h\omega)$$

$$mv = \frac{I_c + mh^2}{I_c}F - m\omega u$$

$$I_c\omega = -Fh$$

In accordance with the example condition, we set $v = \varepsilon V, |V| \le C, \varepsilon \ll 1.$

$$u = \omega(\varepsilon V + h\omega)$$

$$\varepsilon V = \frac{I_c + mh^2}{mI_c}F - \omega u$$

$$I_c\omega = -Fh$$

Let $\varepsilon = 0$, then

$$mu = mh\omega^2 + O(\varepsilon)$$

$$F = \frac{mI_c}{I_c + mh^2}\omega u + O(\varepsilon)$$

$$(I_c + mh^2)\omega = -mh\omega u + O(\varepsilon)$$

$$v = 0 + O(\varepsilon)$$