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Black swan and curvature in an autocatalator model

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Abstract

The aim of this paper is to extend canard constructing method given by J. Ginoux, J. Llibre, and L. Chua [1,2]. In this paper, we consider a black swan construction for three-dimensional autocatalator model using modified flow curvature method.

Keywords: dynamical systems, integral manifolds, autocatalator model, black swans, flow curvature method

1. Introduction

Let us consider an autonomous dynamical system:

$$\begin{cases} \dot{x} = f(x, y, \varepsilon), \\ \dot{y} = \dfrac{1}{\varepsilon} g(x, y, \varepsilon), \end{cases} \tag{1}$$

where x and y are functions of time, ε is a small positive parameter, and the dot refers to differentiation with respect to time t. Assume that $f : \mathbb{R}^{n+m} \to \mathbb{R}^n$ and $g : \mathbb{R}^{n+m} \to \mathbb{R}^m$ are sufficiently smooth.

Let $V \in \mathbb{R}^{n+m}$ be the velocity vector of the point $X = (x, y)$. According [3, p.168] the curve solution to dynamical system (1) would be associated with the coordinates of a point X at the instant t.

Then we have

$$\dot{X} = V(X),$$

with

$$V(X) = \left\{f_1, \ldots, f_n, g_1, \ldots, g_m\right\}.$$

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Let a smooth function $\varphi(X) : \mathbb{R}^{n+m} \to \mathbb{R}$ be given by

$$\varphi(X) = \left| V \wedge \dot{V} \wedge \ddot{V} \wedge \cdots \wedge \overset{(n+m-1)}{V} \right|.$$

Then the manifold $\Omega \subset \mathbb{R}^{n+m}$ is called a *flow curvature manifold* associated with system (1) if $\varphi(X) = 0$ for all $X \in \Omega$ [2,4].

It is obvious that

$$\varphi(X) = \begin{vmatrix} V_1 & \dots & V_{n+m} \\ \dot{V}_1 & \dots & \dot{V}_{n+m} \\ \vdots & & \vdots \\ \overset{(n+m-1)}{V_1} & \dots & \overset{(n+m-1)}{V_{n+m}} \end{vmatrix},$$

therefore, we have

$$\varphi(X) = \begin{vmatrix} \dot{X}_1 & \dots & \dot{X}_{n+m} \\ \ddot{X}_1 & \dots & \ddot{X}_{n+m} \\ \vdots & & \vdots \\ \overset{(n+m)}{X_1} & \dots & \overset{(n+m)}{X_{n+m}} \end{vmatrix}. \tag{2}$$

J. G. Darboux [5] proved that the flow curvature manifold is invariant surface of system (1). Since the Lie derivative $L_X\varphi = 0$, we have that φ is the first integral of (1) [6]. According these we obtain that $\varphi(X)$ is globally invariant. Thus, we have

$$d\varphi = 0. \tag{3}$$

We will return to this equation later.

2. Autocatalator model

A model of a three-dimensional autocatalator has the form [7–9]:

$$\begin{cases} \dfrac{dx}{dt} = \mu\left(\dfrac{5}{2} + y\right) - xz^2 - x, \\ \dfrac{dy}{dt} = z - y, \\ \dfrac{dz}{dt} = \dfrac{1}{\varepsilon}\left(xz^2 + x - z\right), \end{cases} \tag{4}$$

where $\varepsilon > 0$ is a small parameter, $0 \le \mu < 1$, and $x, y, z \in \mathbb{R}^+$.

A *slow surface* of system (4) is the manifold described by equation (Fig.1)

$$x = x_0(y, z) = \frac{z}{1 + z^2}.$$

The slow surface consists of stable and unstable domains. A set of points of the slow surface is called *breakdown surface* if it separates stable and unstable domains. Let we have an additional scalar parameter in the differential system (1). Then, in some special cases [see10, ch.8], we can to glue the stable and unstable domains at one point of the breakdown surface.

A trajectory of the singularly perturbed system (1) is called *canard* if at first moves along the stable slow invariant manifold and then continue for a while along the unstable slow invariant manifold [10, ch.8]. This means that the canard may be a result of gluing stable and unstable slow invariant manifolds at one point of the breakdown surface. This approach was first proposed in [11,12] and was then applied in [13–24].

The term ”canard” (or *duck-trajectory*) was originally introduced by French mathematicians [25–27].