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Hysteretic nonlinearity and unbounded solutions in oscillating systems

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Abstract

The paper is focused on the study of dynamics of oscillating systems described by differential equations with added hysteretic nonlinearities in the case when a hysteretic loop bypass clockwise. For the model of a harmonic oscillator with hysteretic external force, the conditions for existence of unbounded solutions are inferred from a theorem. For the model of a harmonic oscillator with viscous and Coulomb frictions, under external hysteretic perturbations, a set of conditions for the self-oscillation mode are obtained.

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Keywords: unbounded solutions; hysteresis; viscous and Coulomb friction

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1. Introduction

Development of mathematical models of hysteretic phenomena has been stimulated by a wide range of applied problems, mainly those referring automatic control (e.g. [1] and the references therein). In these problems the sources with hysteretic properties cannot be considered separately, but should be considered as a part of a complex system [2,3]. Thus, the hysteretic nonlinearity should be taken into account in the modeling process and should be included in the system of differential equations describing the corresponding dynamical systems.

Investigation of hysteretic phenomena has a long history. Systems with hysteretic blocks were investigated A.A. Andronov back in 1946. The rigorous mathematical theory of hysteresis was formulated in the monograph by M.A. Krasnoselskii and A.V. Pokrovskii [4] where the hysteretic phenomena are considered in the framework of the system's theory. In this monograph, the hysteretic converter is treated as an operator (operator which parametrically depends on its initial state) defined in a corresponding functional space (for example, in the space of continuous functions).

A study of the resonance in the systems with hysteresis is believed to be particularly important [5]. A dynamics of an oscillator with strong nonlinearity was considered in [6], where the phase portrait and trajectory of the oscillator was studied. It was shown that the form of periodic solutions depends on the "origin" of strong nonlinearity. It was concluded that, for the class of equations describing harmonic oscillations with resonance external force and hysteretic operator in the right-hand side of the equations, the presence or absence of unbounded solutions depend on the amplitude of the external force.

In this paper we focus on the resonance properties of the systems in which the energy pumping is attributed to the presence of hysteresis level. Examples of such systems are the oscillations of a ferromagnetic ball in a magnetic field, oscillations of a system of coupled oscillators when the "connection force" has a hysteretic nature [7]. It should be pointed out that these systems can be considered as effective models in the solid state physics. As an example, we can mention modeling of interatomic interactions taking into account various types of dislocations arising due to material's aging. Also, this model can be applied to problems with viscoelasticity.

2. Oscillating system with hysteretic nonlinearity

Let us consider a system the dynamics of which is described by the following equation with the corresponding initial conditions:

$$\begin{aligned} &x + \omega^2 x = R[\omega_0]x, \\ &x(0) = x_0, \quad \dot{x}(0) = x_1, \end{aligned} \tag{1}$$

where R is a non-ideal relay operator with the negative spin and ω_0 is the initial state of the operator (see Fig.1).

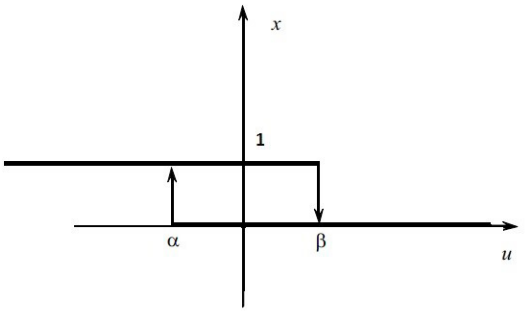

Fig. 1. Non-ideal relay with negative spin.

Theorem: Let the initial value satisfy the condition $x_0 \notin [\alpha;\beta]$. Then the corresponding solutions are unbounded.

Proof: Let us assume that the initial conditions satisfy the following inequality $x_0 > \beta$. In this case, at a certain